MATH 205: Homework 3
Due Wednesday Oct 18

Problems are from Boller and Sally. I recommend at least thinking about all the exercises, even if they are not assigned, as you read through the textbook. I also recommend trying to prove all the Theorems which are left unproven in the book.

Problem 1. Exercises 5.10.3, 5.10.6, 5.10.7, 5.10.8

Problem 2. Suppose $E \subset \mathbb{R}^n$ is a bounded set such that, for all $\varepsilon > 0$ there exists an open set $U \supset E$, with $\partial U$ having measure zero, such that $|U| \leq \varepsilon$. Show that $E$ has measure zero.

Problem 3. Let $R$ be a closed bounded rectangle. and $f : R \to \mathbb{R}$ bounded.
(a) Suppose $f : R \to \mathbb{R}$ bounded. Show that if $E = \{x \in R : f \neq 0\}$ has measure zero then $f$ is Riemann integrable on $R$ and,

$$\int_R f \, dx = 0.$$

(b) Suppose that $f, g : R \to \mathbb{R}$ are bounded and integrable. Show that if $D = \{x \in R : f(x) \neq g(x)\}$ has measure zero then,

$$\int_R f \, dx = \int_R g \, dx.$$

(c) Show that if $f : R \to \mathbb{R}$ is Riemann integrable and $\Omega \subset R$ is an open set with $\partial \Omega$ having measure zero then,

$$\int_\Omega f \, dx = \int_\Omega^\Omega f \, dx = \int_\Omega^\Omega f \, dx.$$

Problem 4. Cylindrical coordinates $(r, \theta, z)$ for $\mathbb{R}^3$ are defined by $(x, y, z) = (r \cos \theta, r \sin \theta, z)$. Spherical coordinates were defined in the previous problem 5.10.6. Compute the following integrals by choosing a convenient coordinate system.

(a) $\int_D z^2$ where $D$ is the hemisphere $D = \{x^2 + y^2 + z^2 \leq 1\} \cap \{z \geq 0\}$.

(b) $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} xy \, dz \, dy \, dx$ (what region is this integral over?)

Problem 5. Exercises 6.1.10, 6.1.11, 6.1.13