Math 6070-1, Spring 2014; Assignment #5

Assigned on: Friday February 28, 2014
Due: Monday March 10, 2014

1. Read the module on χ^2 tests (`http://www.math.utah.edu/~davar/math6070/2014/Chisquared.pdf`).

2. A random sample of 100 people from a certain population resulted in the following:

Age Group	**No. of samples**
0–16	27
17-24	26
25–34	15
35–49	22
50–100	10

   Perform a χ^2 test in order to see if the data is distributed uniformly across the mentioned age groups.

3. Consider two finite populations: One has respective proportions $\theta_1, \ldots, \theta_m$ for its individuals of types $1, \ldots, m$. The other has respective proportions $p_1, \ldots, p_m$ for its individuals of type $1, \ldots, m$. Let $\boldsymbol{\theta} := (\theta_1, \ldots, \theta_m)'$ and $\boldsymbol{p} := (p_1, \ldots, p_m)'$ be the respective probability vectors. We assume that $\boldsymbol{p}$ and $\boldsymbol{\theta}$ are unknown.

   Independent samples are taken from each population [independently from one another]. Let the sample sizes be n_1 and n_2 respectively, and denote by $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{p}}$ the sample-proportion vectors of types.

   (a) Prove that $\hat{\boldsymbol{\theta}}$ converges to $\boldsymbol{\theta}$ in probability in the following sense:

   $$\left\|\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}\right\| \xrightarrow{\text{P}} 0 \quad \text{as } n_1 \to \infty.$$

   This, of course, would also prove that $\hat{\boldsymbol{p}}$ converges to $\boldsymbol{p}$ in probability as $n_2 \to \infty$, since the problem is symmetric in the two populations.

   (b) Prove that the random vector $\sqrt{n_1}\{\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\}$ has a limiting distribution, as $n_1 \to \infty$. Identify that limiting distribution. Perform the analogous analysis for $\sqrt{n_2}\{\hat{\boldsymbol{p}} - \boldsymbol{p}\}$ [you do not need to reproduce the work; just work out the statement].

(c) Consider the null hypothesis, $H_0 : \boldsymbol{\theta} = \boldsymbol{p}$ against its two-sided alternative. Describe a condition under which you can ensure that the distribution of $\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{p}}$ as a large-sample asymptotically normal approximation. Carefully state your central limit theorem.

(d) Use your central limit theorem to devise a χ^2 test for $H_0 : \boldsymbol{\theta} = \boldsymbol{p}$.