3.15. No, $\mathscr{A}$ need not be an algebra. For instance, let μ be the counting measure on positive integers. Then $E \in \mathscr{A}$ and $(D\mu)(E) = 1/2$, where E denotes all even integers. Now let F be the collection of all positive integers m such that: (i) If $2^k < m \leq 2^{k+1}$ for some even integer $k \geq 0$ then m is even; (ii) else m is odd. Then $F \in \mathscr{A}$ and $(D\mu)(F) = 1/2$, but $E \cap F \notin \mathscr{A}$.

Let μ be counting measure on positive integers and A the set of all even integers. Evidently, $(D\mu)(A) = \frac{1}{2}$. However, $(D\mu)(\{x\}) = 0$ for all x so that $(D\mu)(A) \neq \sum_{x \text{ even integer}} (D\mu)(\{x\})$. Therefore, $D\mu$ is not countably additive on $\mathscr{A}$.