3.1. Because $\Omega \in \mathscr{F}_\alpha$ for all $\alpha \in A$, $\Omega \in \cap_{\alpha \in A} \mathscr{F}_\alpha$. If $E \in \cap_{\alpha \in A} \mathscr{F}_\alpha$ then $E \in \mathscr{F}_\alpha$ for all $\alpha \in A$. This proves that $E^c \in \mathscr{F}_\alpha$ for all $\alpha \in A$, whence $E^c \in \cap_{\alpha \in A} \mathscr{F}_\alpha$. It, therefore, remains to prove that whenever $E_1, E_2, \ldots$ are in $\cap_{\alpha \in A} \mathscr{F}_\alpha$ then so is $\cup_{n=1}^{\infty} E_n$. But $\cup_{n=1}^{\infty} \in \mathscr{F}_\alpha$ for all α, whence the claim.

To finish, suppose $\mathscr{A}$ is an algebra. We can then define $\mathscr{F} = \cap \mathscr{G}$, where the intersection is taken over all σ-algebras $\mathscr{G}$ that contain $\mathscr{A}$. This is clearly the right object. Note that the said intersection is not empty because the power set is a σ-algebra that contains $\mathscr{A}$.