

Recall that for all $z \geq 0$,

$$e^{-z} = 1 - z + \varepsilon(z),$$

where $\varepsilon(z) \sim z^2/2$ as $z \rightarrow 0$. [This follows from the Taylor expansion with remainder, for example.] Therefore, as $t \rightarrow 0$,

$$\int_0^t e^{-x^2} dx = \int_0^t (1 - x^2 + \varepsilon(x^2)) dx = t - \frac{t^3}{3} + \int_0^t \varepsilon(x^2) dx = t - \frac{t^3}{3} + \frac{\theta(t)}{2} \int_0^t x^4 dx = t - \frac{t^3}{3} + \frac{t^5 \theta(t)}{10},$$

where $\theta(t) \rightarrow 1$ as $t \rightarrow 0$. The exercise follows.