2.6. Let $F_n(x) := \mathrm{P}\{X_n \le x\}$ and $F(x) := \mathrm{P}\{X \le x\}$. F_n and F are non-decreasing, right-continuous, 1 at ∞, and 0 at $-\infty$. Choose and fix $\varepsilon > 0$. We can find points $-\infty < x_0 < x_1 < \ldots < x_m < \infty$ such that: (a) $0 \le F(x_{i+1}) - F(x_i) \le \varepsilon$; and (b) $F(x_0) + F(x_m) \le \varepsilon$. Evidently, $\lim_{n\to\infty} \max_{0\le j\le m} |F_n(x_j) - F(x_j)| = 0$. Therefore, there exists n_0 such that for all $n \ge n_0$ and $j = 0, \ldots, m$, $F(x_j) - \varepsilon \le F_n(x_j) \le F(x_j) + \varepsilon$. If $x \in [x_j, x_{j+1}]$ for some $j = 0, \ldots, m-1$ then for all $n \ge n_0$, $F(x) - 2\varepsilon \le F(x_j) - \varepsilon \le F_n(x_j) \le F_n(x) \le F_n(x_{j+1}) \le F(x_{j+1}) + \varepsilon \le F(x) + 2\varepsilon$. This proves that

$$\lim_{n\to\infty} \sup_{x\in[x_0,x_m]} |F_n(x) - F(x)| = 0.$$

On the other hand, for all $x \le x_0$ and $n \ge n_0$, $F_n(x) \le F_n(x_0) \le F(x_0) + \varepsilon \le 2\varepsilon$. Likewise, for all $x \ge x_m$ and $n \ge n_0$, $F_n(x), F(x) \le 2\varepsilon$. Thus, $F_n \to F$ uniformly.