We can simplify this by writing $k^2 = k(k-1)+k$. This leads us to:

$$\begin{aligned}
\mathrm{E}(X^2) &= \binom{r+b}{n}^{-1} \sum_{k=2}^{n} k(k-1)\binom{r}{k}\binom{b}{n-k} + \mathrm{E}X \\
&= \binom{r+b}{n}^{-1} \sum_{k=2}^{n} \frac{r!}{(k-2)!(r-k)!}\binom{b}{n-k} + \frac{nr}{b+r}.
\end{aligned}$$

But

$$\begin{aligned}
\sum_{k=2}^{n} \frac{r!}{(k-2)!(r-k)!}\binom{b}{n-k} &= r(r-1)\sum_{k=2}^{n}\binom{r-2}{k-2}\binom{b}{\{n-2\}-\{k-2\}} \\
&= r(r-1)\binom{b+r-2}{n-2}.
\end{aligned}$$

Therefrom,

$$\mathrm{E}(X^2) = r(r-1)\frac{\binom{b+r-2}{n-2}}{\binom{b+r}{n}} + \frac{nr}{b+r} = \frac{rn(r-1)(n-1)}{(b+r)(b+r-1)} + \frac{nr}{b+r} = \frac{nr}{b+r}\left[\frac{(r-1)(n-1)}{b+r-1}+1\right],$$

and

$$\mathrm{Var}X = \mathrm{E}(X^2) - (\mathrm{E}X)^2 = \frac{nr}{b+r}\left[\frac{(r-1)(n-1)}{b+r-1} + 1 - \frac{nr}{b+r}\right].$$