26. Conditionally on $U = u$, $X = \text{Bin}(n, u)$. Therefore, for all $x = 0, \ldots, n$,

$$p_X(x) = \int_0^1 \mathrm{P}\{X = x \,|\, U = u\}\, du = \int_0^1 \binom{n}{x} u^x (1-u)^{n-x}\, du = \binom{n}{x} \frac{\Gamma(x+1)\Gamma(n-x+1)}{\Gamma(n+2)} = \binom{n}{x} \frac{x!(n-x)!}{(n+1)!} = \frac{1}{n+1}.$$