1.13. For all $a, b \geq 0$ consider

$$J(a,b) := \int_{-b}^{a} \frac{x}{1+x^2}\,dx.$$

Evidently, $J(a,0) = \ln(1+a^2)$. Therefore,

$$J(a,b) = \ln\left(\frac{1+a^2}{1+b^2}\right).$$

Therefore, $J(a,a) = 0$, whereas $\lim_{a\to\infty} J(a,a^2) = -\infty$. This proves that $\int_{-\infty}^{\infty} x/(1+x^2)\,dx$ is not well defined. But the latter, if well defined, would be $\pi \mathrm{E}X$.

To finish, consider a random variable Y whose density function is $f_Y(y) := y^{-2}$ $(y > 1)$. In this case, $\mathrm{E}Y = \int_1^{\infty} y^{-1}\,dy = \infty$ is well-defined but infinite.