Math 5080–1
Solutions to homework 1

May 22, 2012

 # 18, p. 86.

(a) If $a \le 0$, then $F(a) = 0$. If $0 \le a \le 3$, then

$$F(a) = \int_0^a \frac{2x}{9}\, dx = \frac{a^2}{9}.$$

If $a > 3$, then $F(a) = 1$.

(b) $P\{X \le 2\} = F(2) = 4/9$.

(c) $P\{-1 < X < 1.5\} = P\{-1 < X \le 1.5\} = F(1.5) - F(-1)$. Now $F(1.5) = (3/2)^2/9 = 1/4$, and $F(-1) = 0$. Therefore, $P\{-1 < X < 1.5\} = 1/4$.

(d) $P\{X \le m\} = F(m)$ and $P\{X \ge m\} = 1 - F(m)$ because $P\{X = m\} = 0$. Therefore, we seek to find a number m such that $F(m) = 1 - F(m)$; that is, $F(m) = 1/2$. Solve: $m^2/9 = 1/2$ to deduce that $m^2 = 9/2$ and hence $m = 3/\sqrt{2}$.

(e) $E(X) = \int_0^3 af(a)da = (2/9)\int_0^3 a^2 da = 2$.

 # 26, p. 87.

(a) $E(X) = (1 \times 0.3) + (2 \times 0.3) + (3 \times 0.2) + (4 \times 0.1) = 1.9$.

(b) $E(X^2) = (1 \times 0.3) + (4 \times 0.3) + (9 \times 0.2) + (16 \times 0.1) = 4.9$. Therefore, $\mathrm{Var}(X) = 4.9 - 1.9^2 = 1.29$.

(c) The owner's net profit is $Y = 35X - 40$; therefore the expected net profit is $E(Y) = 35E(X) - 40 = (35 \times 1.9) - 40 = 26.5$ dollars. Also, $\mathrm{Var}(Y) = 35^2\mathrm{Var}(X) = 1580.25$ squared dollars.

 # 33, p. 88. We follow the discussion of pp. 77–78, and write $H(x) := e^x$ as $H(x) \simeq H(\mu) + H'(\mu)(x-\mu) + \frac{1}{2}H''(\mu)(x-\mu)^2$, where $H(\mu) = e^\mu$, $H'(\mu) = e^\mu$ and $H''(\mu) = e^\mu$. Therefore, the approximate mean is given by $E(e^X) \simeq H(\mu) + \frac{1}{2}H''(\mu)\sigma^2 = e^\mu\{1 + (\sigma^2/2)\}$ and the approximate variance is given by $\text{Var}(e^X) \simeq [H'(\mu)]^2\sigma^2 = e^{2\mu}\sigma^2$; see **(2.4.18)** and **(2.4.29)** on p. 78.

 # 34, p. 88.

(a) X takes the respective values 1, 2, and 5 with respective probabilities $1/8$, $1/4$, and $5/8$.

(b) $1/4$.

 # 36, p. 88.

(a) We merely follow along the definition of MGFs:

$$\begin{aligned} M_X(t) &= E(e^{tX}) = \int_{-2}^{\infty} e^{tx} e^{-(x+2)}\, dx \\ &= e^{-2} \int_{-2}^{\infty} e^{x(t-1)}\, dx. \end{aligned}$$

If $t \geq 1$, then the preceding is infinity. Else, if $-\infty < t < 1$, then $M_X(t) = e^{-2+2(1-t)}/(1-t) = e^{-2t}/(1-t)$.

(b) For all $-\infty < t < 1$,

$$M_X'(t) = \frac{-2(1-t)e^{-2t} + e^{-2t}}{(1-t)^2} = \frac{(2t-1)e^{-2t}}{(1-t)^2} \quad \Rightarrow \quad E(X) = M_X'(0) = -1.$$

Also,

$$M_X''(t) = \frac{(1-t)^2\left\{-2(2t-1)e^{-2t} + 2e^{-2t}\right\} + 2(2t-1)e^{-2t}(1-t)}{(1-t)^4} \Rightarrow E(X^2) = M_X''(0) = 2.$$