

**Example 20.1.** Suppose that the offspring mass function is given by

$$f(k) = \begin{cases} 1/4 & \text{if } k = 0, \\ 1/4 & \text{if } k = 1, \\ 1/2 & \text{if } k = 2. \end{cases}$$

Then,  $G(s) = \frac{1}{4} + \frac{1}{4}s + \frac{1}{2}s^2$ , and hence  $G(s) = s$  is the same equation as

$$2s^2 - 3s + 1 = 0.$$

The solutions are

$$s = \frac{3 \pm \sqrt{9 - 8}}{4} = \frac{1}{2} \text{ and } 1.$$

Thus, the probability of ultimate extinction is  $1/2$ .

... examples of mgf's