Solutions to Midterm 1
Mathematics 5010–1, Summer 2009

1. *A fair coin is tossed 120 times. If all possible outcomes are equally likely, then what is the probability that there are no heads tossed?*

   Solution: Here, Ω is the collection of all possible ways to write down 120 H's and T's. By the principle of counting, Ω has $2^{120} \approx 1.33 \times 10^{36}$ elements. Each element of Ω has probability 2^{-120} of getting selected; only one of them is all T's. Therefore, the answer is $2^{-120} \approx 7.52 \times 10^{-37}$.

2. *There are 50 men and 50 women in a room. You select 4 at random and independently [i.e., sampling with replacement]. What is the probability that the number of men in the sample is the same as the number of the women in the sample?*

   Solution: This is a simplified version of Problem 8 [p. 128, see homework 1]. You apply Problem 18 (page 128) with $p = q = 1/2$ and $n = 2$ to obtain

   $$\binom{2n}{n}(pq)^n = \binom{4}{2}4^{-2} = \frac{4!}{2! \cdot 2!} \times \frac{1}{16} = \frac{3}{8}.$$

3. *Four digits are selected independently at random (without repetition) from $\{0, \ldots, 9\}$. What is the probability the the four digits form a run? [For example, 0, 1, 2, 3.]*

   Solution: This is Problem 3(a) [p. 126, see homework 1].

4. *How many different messages can be sent by using five dashes and three dots?*

   Solution: $\binom{8}{3}$.