Math 2270-2, Quiz 1

Tuesday May 31, 2016

1. (10 points) Which of the following is in reduced row echelon form? Circle the pivots and identiy the pivoting columns in the case that a matrix is in reduced echelon form.

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \boldsymbol{C} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Solution. The (3 ,3) entry of $\boldsymbol{A}$ is not 1; so $\boldsymbol{A}$ is not in reduced echelon form. The (2 ,3) entry of $\boldsymbol{B}$ is not zero, so $\boldsymbol{B}$ is also not in reduced echelon form. The (1 ,4) entry of the matrix $\boldsymbol{C}$ is not zero. So $\boldsymbol{C}$ is not in reduced row echelon form.

2. (5 points) Consider the vectors

$$\boldsymbol{a} = \begin{pmatrix}1\\1\\1\end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix}1\\1\\-2\end{pmatrix}, \quad \boldsymbol{c} = \begin{pmatrix}-1\\0\\-2\end{pmatrix}, \quad \boldsymbol{d} = \begin{pmatrix}1\\2\\3\end{pmatrix}, \quad \boldsymbol{e} = \begin{pmatrix}0\\0\\1\end{pmatrix}.$$

Is $\boldsymbol{e}$ a linear combination of the vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$, and $\boldsymbol{d}$? Justify your answer.

Solution. We wish to find numbers—not all zero—x_1, x_2, x_3, x_4 such that $x_1\boldsymbol{a} + x_2\boldsymbol{b} + x_3\boldsymbol{c} + x_4\boldsymbol{d} = \boldsymbol{e}$. Let us row-reduce the augmented matrix

$$\begin{pmatrix}1 & 1 & -1 & 1 & 0\\ 1 & 1 & 0 & 2 & 0\\ 1 & -2 & -2 & 3 & 1\end{pmatrix}.$$

Replace (row 2) by (row 2 − row 3) and (row 3) by (row 1 − row 3):

$$\begin{pmatrix}1 & 1 & -1 & 1 & 0\\ 0 & 3 & 2 & 5 & -1\\ 0 & 3 & 1 & -2 & -1\end{pmatrix}.$$

Replace (row 3) by (row 2 − row 3):

$$\begin{pmatrix}1 & 1 & -1 & 1 & 0\\ 0 & 3 & 2 & 5 & -1\\ 0 & 0 & 1 & 7 & -2\end{pmatrix}.$$

Replace (row 2) by $\frac{1}{3}\{$(row 2)$-2\times$(row 3)$\}$ and (row 1) by (row 1)+(row 3):

$$\begin{pmatrix}1 & 1 & 0 & 8 & -2\\ 0 & 1 & 0 & -3 & 1\\ 0 & 0 & 1 & 7 & -2\end{pmatrix}.$$

Replace (row 1) by (row 1)−(row 2):

$$\begin{pmatrix}\boxed{1} & 0 & 0 & 11 & -3\\ 0 & \boxed{1} & 0 & -3 & 1\\ 0 & 0 & \boxed{1} & 7 & -2\end{pmatrix}.$$

We can now see that x_4 is a free variable, and

$$\begin{aligned} x_1 \qquad\qquad\quad &+11x_4 = -3\\ x_2 \qquad\quad &-3x_4 = 1\\ x_3 \quad &+7x_4 = -2. \end{aligned}$$

To obtain a solution, set $x_4 = 0$ to see that $x_1 = -3$, $x_2 = 1$, and $x_3 = -2$.