Sec 8.1 (Sequences)

What is a sequence?

Def : A Sequence is a list of numbers given in a specific order

Ex { 1, 45 , 2 , 90,3.14159265, 2.718281828459045, 10 ,-1,0 ,1}

Some times the numbers follow a pattern, In these cases we can usually write a formula for the nth term of a sequence.

Ex {3 , 9 , 27 , 81 , ……}

Sometimes even though the sequences follow a pattern we can't just write the nth term down , an example is the Fibonacci sequence.

{1,1,2,3,5,8,13,21,……}

Def: A recursive sequence is a sequence in which the next term depends on the previous terms.

Sec 8.1 (Sequences)

Finding nth terms:

What is the 5th 8th and 100th term of the sequence $a_n = 1 + \frac{n^2}{n+1}$

Find an expression for the nth term of the sequence {3/4 , 9/5,27/6,....}

Find an expression for the nth term of the sequence { -2/5 , 1/2, -4/7, 5/8,...}

Sec 8.1 (Sequences)

Sequences can have infinitely many terms and sometimes they go to infinity but sometimes they approach a specific value. That is the sequences have a limit.

Ex Consider the sequence defined by:

$$a_n = \frac{2n^2 + n}{4n^2}$$

What value are the nth terms approaching as n approaches infinity?

Lets Test it:

```
ForEach ($number  in 1..20000) {(2*$number*$number+$number)/(4*$number*$number)}
```

If sequence $\{a_n\}$ has limit L this can be written as

$$\lim_{n\to\infty} a_n = L \quad or \quad a_n \to L \quad as \;\; n \to \infty$$

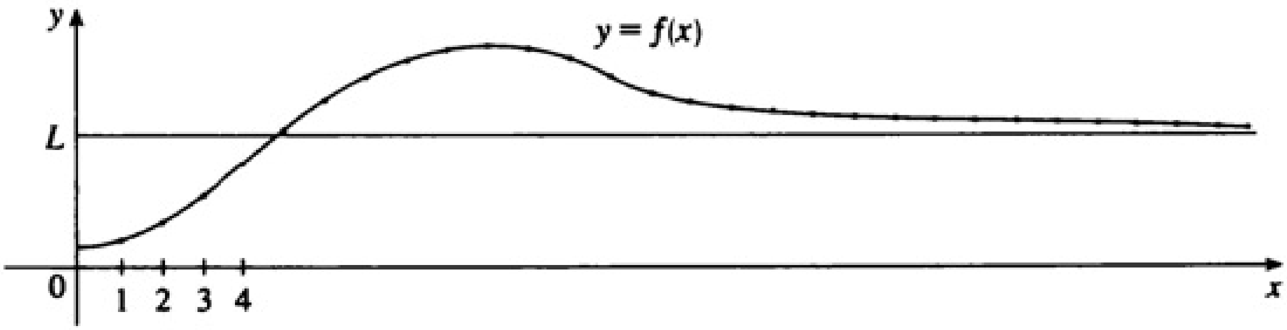
It can be found in the same was as taking limits of functions…….

Ex Find the limit of $a_n = \frac{2n^2+n}{4n^2}$

Sec 8.1 (Sequences)

Theorem: If $\lim_{x\to\infty} f(x) = L$ *and* $f(n) = a_n$ *when n is an integer then* $\lim_{n\to\infty} a_n = L$

This just means the limit of a sequence like $a_n = \frac{3n}{n-1}$ IS the same as the limit of $f(x) = \frac{3x}{x-1}$



As such all of the usual limit properties apply:

If $\{a_n\}$ and $\{b_n\}$ are convergent sequences and c is a constant, then

$$\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} (a_n - b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n \qquad \lim_{n\to\infty} c = c$$

$$\lim_{n\to\infty} (a_n b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} \quad \text{if } \lim_{n\to\infty} b_n \neq 0$$

$$\lim_{n\to\infty} a_n^p = \left[\lim_{n\to\infty} a_n\right]^p \quad \text{if } p > 0 \text{ and } a_n > 0$$

Sec 8.1 (Sequences)

Limit of a sequence inside a function

If $\lim_{n\to\infty} a_n = L \quad and\ \ f(x)\ is\ a\ continuious\ function\ \ then\ \ \lim_{n\to\infty} f(a_n) = f(L)$

Ex Find $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Sec 8.1 (Sequences)

The squeeze theorem:

If $a_n \leq \; c_n \leq b_n \;\; and \;\; \lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = L \;\; Then \;\; \lim_{n\to\infty} c_n = L$

Sec 8.1 (Sequences)

Geometric Sequences

A geometric sequence is a sequence where we multiply the previous term by the same number to get the next number. Ex{2 , 4, 8 , 16...} {1/5, 1/25,1/125,...} These sequences have what is called a common ratio r defined as the ratio between two successive terms.

Ex What is the common ratio for the geometric sequence {1 , 2/3, 4/9, 8/27,...}

Find an expression for the nth term of the geometric sequence given above.

The Nth term of a geometric sequence can be written as $a_n = a_1 r^n \quad n = 0,1,2,3\ldots$ where a_1 is the first term in the sequence and r the common ratio

Sec 8.1 (Sequences)

When Does a Geometric sequence converge?

As you have seen with our infinity screen when I start up google hangouts to record our lectures the screen size in each subsequent window is decreased by about 40%. Find an expression for the area of the nth window assuming we define the first full screen to have area 1. Would this sequence converge?

Sec 8.1 (Sequences)

Monotonic sequences: A monotonic sequence is one that is only increasing or decreasing.

Bounded Sequences:

Definition A sequence $\{a_n\}$ is **bounded above** if there is a number M such that

$$a_n \leqslant M \qquad \text{for all } n \geqslant 1$$

It is **bounded below** if there is a number m such that

$$m \leqslant a_n \qquad \text{for all } n \geqslant 1$$

If it is bounded above and below, then $\{a_n\}$ is a **bounded sequence**.

Sec 8.1 (Sequences)

Monotonic Sequence Theorem

Every bounded monotonic sequence is convergent!