

Comments on the Mathcircle:

- (1) I cut the first sheet into 3 parts at the spaces. I think this served as a nice break from parts 1 and 2. Also Problem 5 in “Getting a better feel” kind of gives away Problem 3 in “Getting a better feel”.
- (2) I ran this math circle over an hour and a half (really more like an hour and ten minutes) with high school and middle school students. Some got to the second page, but I don't think anyone made much progress on it.
- (3) A hint on Problem 1 in “Getting started”. You can tell that k is a 3 digit number, because 4 and higher digit numbers have that f decreases their number of digits. Indeed, $m \cdot 9^2 < 10^{m-1}$ if $m \geq 4$. Once you are at 3 digit numbers, the highest you can reach is $243 = 3 \cdot 81$. But then you can only reach $4 + 2 \cdot 81 = 166$. Then you can only reach $1 + 36 + 81$ and so forth. Following this reasoning $k = 99$.
- (4) A hint on Problem 3 in “Getting started”. A terminating sequence needs to include a number less than 99 by Problem 1 in “Getting started”. Because $f(n) \leq 162 = f(99)$ for all $n \leq 162$, once you get below 99 you never get above 162. So the eventual period is at most 162.
- (5) I don't have slick proofs of Problems 4 and 5 of “Getting started”, but the process of elimination is not so bad. For example, In problem 4 it's not hard to reduce to 2 digit numbers (by Problem 1). Now for each chunk of ten numbers $d \cdot 10 + 0$ to $d \cdot 10 + 9$, it is easy to eliminate them.
- (6) When I ran the math circle some questions that came up: Is every number happy for some f_m ? Are there infinitely many happy numbers not of the form 10^k ?