Homework 2

For this homework you should give classification for continuous maps from S^1 to itself similar to Theorem 3.1 in the notes. There is more than one was to do this. We working on some ideas in class that I'll outline here. However, you don't have to follow this outline.

First we describe S^1 in complex coordinates as

$$S^1 = \{z \in \mathbb{C} | |z| = 1\}$$

and then defined maps $f_n \colon S^1 \to S^1$ by $f_n(z) = z^n$. We then want to show that every continuous $f \colon S^1 \to S^1$ is homotopic to a unique f_n.

Note that the maps f_n have the extra property that $f_n(1) = 1$ (in complex coordinates). Can you show that every map is homotopic to a map that takes 1 to itself? (Hint: Rotations)

The approach we discussed in class was to try to mimic the proof of Theorem 3.1. To do that we need new versions of Propositions 3.2 and 3.9.

Here's the replacement we came up with for Proposition 3.2:

Proposition 0.1 *Let*

$$f \colon (S_1, 0) \to (S^1, 0)$$

be a continuous map of pairs. Then there exist a unique continouous map

$$\tilde{f} \colon (\mathbb{R}, 0) \to (S^1, 0)$$

such that $f \circ p = p \circ \tilde{f}$.

Here's our replacement for Proposition 3.9:

Proposition 0.2 *Let*

$$F \colon (S_1 \times [0,1], (1,0)) \to (S^1, 0)$$

be a continuous map of pairs. Then there exist a unique continouous map

$$\tilde{F} \colon (\mathbb{R} \times [0,1], 0) \to (S^1, 0)$$

such that $F \circ p = p \circ \tilde{F}$.

You need to prove both of these propositions. You can do this using Propositions 3.2 and 3.9. One possible first step is to prove a smaller extension of Proposition 3.2. For example in 3.2 we could replace the interval $[0,1]$ with the interval $[-n,n]$. Can you use Proposition 3.2 to prove this extension?

Once you've proved this two new propositions proof of the classification theorem is very similar to the proof Theorem 3.1 from the notes. One new complication that will arise is the following: Given a continuous map $\tilde{f} \colon \mathbb{R} \to \mathbb{R}$ when is there a map $f \colon S^1 \to S^1$ with $f \circ p = p \circ \tilde{f}$?