\[ p: (E, e_\circ) \Rightarrow (B, b_0) \]
\[ e_\circ \in \text{p}^{-1}(b_0) \]

Show that \( \exists \ p_1 : E \Rightarrow E \) s.t.
\[ p_1(e_\circ) = e_0 \quad \& \quad \text{p} \circ p_1 = \text{id} \]

1
\[ p_1 \Rightarrow (E, e_\circ) \]
\[ \text{p} \Rightarrow (E, e_\circ) \]
\[ (E, e_\circ) \Rightarrow (B, b_0) \]

2
\[ p_1 \Rightarrow (E, e_\circ) \]
\[ \text{p} \Rightarrow (E, e_\circ) \]
\[ (E, e_0) \Rightarrow (B, b_0) \]

Want to show \( p_0 \circ p_1 = p_1 \circ p_0 = \text{id} \)

\[ \begin{array}{c}
\begin{array}{c}
\vdash \text{id} \quad \text{is such a lift}
\end{array}
\end{array} \]
\[ \begin{array}{c}
\begin{array}{c}
\vdash \text{as is } p_0 \circ p_1
\end{array}
\end{array} \]

**Lemma** Let \( q : E \rightarrow \mathbb{E} \) s.t.
\[ p \circ q = p \quad \& \quad q(x) = x \quad \text{for some } x \in \mathbb{E} \]
\[ \Rightarrow \ q = \text{id}. \]

**Proof** Apply \( \text{Fad lifting lemma}\) to
\[ \begin{array}{c}
\begin{array}{c}
\vdash (E, x) \\
\Rightarrow (E, x)
\end{array}
\end{array} \]
\[ \begin{array}{c}
\begin{array}{c}
\vdash (B, p(x)) \\
\Rightarrow (B, p(x))
\end{array}
\end{array} \]
\( q \) & \( id \) are both left \( \text{homogeneous} \) with \( q(x) = x \) & \( id(x) = x \)

\( C. \) \( q = id. \)

**Cor** Let \( \varphi_0, \varphi_1 : E \rightarrow C \) s.t.
\( \varphi_0 \circ \varphi_1 = p \) \& \( \varphi_0(x) = \varphi_1(x) \) for some \( x \in E \).
\[ \Rightarrow \varphi_0 = \varphi_1 \]

\( P. \)
\[ \begin{array}{c}
\varphi_0, \varphi_1 \ (E, \quad \text{hom}) \\
\downarrow \quad \quad \text{h} \\
(E, \quad x) \quad \text{h} \ (\varphi_1, \ varphi_1(x))
\end{array} \]

Apply previous to \( \varphi_0 \circ \varphi_1^{-1} \).

\[ \varphi_1^{-1} (\varphi_0) \rightarrow \text{homeo} (E) \]
\[ e_i \rightarrow p_i \]

where \( p_i(c_i) = e_o \) \& \( p_{op} = p \).

Show that the image is a \( \text{Subgroup} \) \( G \).

**Comp.** Need to show \( p_1 \circ p_2 \in G \).
\[ p_0 (p_1 \circ p_2) = (p_0 p_1) \circ p_2 = p \circ p_2 = p \]

Need to find \( e \in \text{h} \ dark \ (\varphi_0) \) s.t.
\[ p_1 \circ p_2 (c_0) = e_o \quad \Rightarrow \quad p_1 \circ p_2 = p_o \in G \]
\[ e_0 = p_1^{-1} (e_0) \]

To use this need to know that \( p_o \in G \).
By (C1) we know \( p_o \) is a \( \text{homeo} \).
Also let $e_2 := p_2(e_0)$, then $p_2^{-1}(e_2) = p_2^{-1}(e_0)$.

By lemma $p_2 = p_2^{-1} \Rightarrow y_2 \in G$.

(4) Show that $G$ is a deck action.

UCD is evenly covered if $p^{-1}(U) = U \cup U_a$ where $P|_{U_a}$ is a homeo to $U$.

Pick $x \in E$, assume $U$ is an evenly covered nbhd of $p(x)$ in $B$, let $V$ be the component of $p^{-1}(U)$ that contains $x$. $P|_V$ is injective.

Let $p_i \in G$. Show if $p_i(U) \cap V = \emptyset \implies p_i = id$.

Let $y \in p_i(U) \cap V$. Then $p(p_i(y)) = p(y)$ so $p_i(y) = y$ since $P|_V$ is injective.

$\implies p_i = id$. 
To show \( f \) is well defined & a bijection, need to show that
\[ q(x) = q(y) \iff p(x) = p(y). \]

\[ \Rightarrow \exists \ p, \ x \in \ E, \ \text{such that} \ p(x) = y \]
\[ \Rightarrow p(y) = p(x(x)) = (p \circ p)(x) \]
\[ = p(x). \]

\[ \Leftarrow p(x) = p(y), \ \text{by the iff lemma} \]
\[ \exists \ p, \ x \in \ E, \ \text{such that} \ p(x) = y \] &
\[ p \circ p = p. \]

let \( e_{0} \) = \( p^{-1}(e_{0}) \) = \( p, \in \mathbb{E}. \)