**CLASS EXERCISE**

Classify all pared homotopy classes $f: (\mathbb{S}^{1}, \mathbb{S}^{0}, 1) \to (\mathbb{S}^{1}, \mathbb{S}^{0}, 3, 2, 3)$. 

What are the "model maps" all such $f$ are homotopic to? 

What are the $f_\iota$ in this case? 

To simplify, assume that $f(0) = 10$ and $f(1) = 7 \iota$. 

$\tilde{f}_\iota(t) = (n + \frac{1}{2}) \iota$
\[ f_n(x) = \left\lfloor (n + \frac{1}{2}) x \right\rfloor \quad n \in \mathbb{Z} \]

\[ f_n(0) = \left\lfloor n \right\rfloor = 0 \]

\[ f_n(1) = \left\lfloor \frac{1}{2} + n \right\rfloor = 1 \frac{1}{2} \]

\[ f_n(t) = (n + \frac{1}{2}) t \]
EXISTENCE

Claim: \( \exists x \text{ s.t. } f \leq f_1 \)

By LL, \( \exists f : \mathbb{D} \rightarrow \mathbb{R} \)

Set \( f(\bar{c}) = 0 \) \& \( f = \pi^{-1} \bar{x} \).

\[ \Rightarrow f(\bar{c}) = \{ y \in \mathbb{R} \} = \prod_0 \bar{f}(\bar{c}) \]

\[ \Rightarrow \bar{f}(\bar{c}) \in \pi^{-1}(\{ y \}) \]

\[ \Rightarrow \exists n \text{ s.t. } f_1(\bar{c}) = \bar{x} + n. \]

Guess, that \( f \sim_\rho f_1 \)

\[ \tilde{f}_n(t) = (u + t/\epsilon) \& \]

\[ \tilde{f}_n(0) = u \epsilon \frac{1}{\epsilon} = \tilde{f}(c) \]
Given \( f : S \to S \),

\[ G(s, t) = (1 - \varepsilon) \tilde{f}(s) + \varepsilon \tilde{f}_n(s) \]

by analogy between \( f \) and \( f_n \).

\[ G(s, t) \to \pi \tilde{f}(s, t) \]

**UNIQUENESS**

Give \( f : S \to S \),

with \( f(0) = \varepsilon_0 \) and \( f(N) = \Sigma \varepsilon_i \).

\[ \exists! n \in \mathbb{N} \quad f \preceq f_n \]

**PE strategy:** Assume

\[ f \preceq f_n \quad \& \quad f \preceq \bar{f}_n \]

\[ \Rightarrow \quad f_n \preceq \bar{f}_n \quad \Rightarrow \quad n = m. \]

need to show
\[ \exists \quad \text{a homotopy} \]
\[ g : \{0,1\} \times [0,1] \to S' \]

s.t.
\[ f_0(t) = g_0(t) : G(0, t) \quad \& \quad f_1(t) = g_1(t) : G(1, t) \]

with
\[ G([0,1] \times [0,1]) \subset \{ \{02\}, \{13\} \} \]

\[ G(0,0) = f_0(0) = \{0\} \]
\[ \& \quad g(1,0) = f_1(0) = \{1\} \]

\[ \Rightarrow \quad G([0,1] \times [0,1]) \subset \{ \{02\}, \{13\} \} \]

Both connected \[ \Rightarrow \quad G([0,1] \times [0,1]) \subset \{ \{02\}, \{13\} \} \]

so image must be connected.

By H.L.L \[ \exists! \]
\[ \varphi : \{0,1\} \to 1R \]
with \( \zeta(0,0) = 0 \) &
\[ \zeta(\gamma t) = \prod \zeta(\gamma t). \]

Note \( \tilde{g}_0(\gamma t) \in \tilde{\gamma}(\gamma,0) \) is a lift of \( f_0 \) with
\[ \tilde{g}_0(0) = 0 = \tilde{g}(0,0). \]
By the uniqueness in L.L \( \Rightarrow \) \( \tilde{g}_0 = f_0 \)
\[ = f_1 \tilde{g}_0(\gamma t) = f_0(\gamma t) = \gamma t + \gamma \]

we know
\[ \{ \tilde{g}(0,1) \} \leq \zeta(\gamma \xi(0,1)) = \prod \zeta(\gamma \xi(0,1)) \quad \text{dirichlet's } \]

\[ \Rightarrow \tilde{g}(\gamma \xi(0,1)) \leq \prod^{-1}(\gamma \xi(0,1)) = \{ 1, 2, 3, 5, 7, 11, 13, 17 \} \]
Also \( \tilde{g}(0,0) > 0 \) \( \Rightarrow \) \( \tilde{g}(\gamma \xi(0,1)) = \{ \} \)
\[ \tilde{g}_1(\gamma t) \in \tilde{\gamma}(\gamma,1) \] is a lift of \( f_1 \)
\[ \tilde{g}_1(c) = 0 \quad \Rightarrow \quad \tilde{g}_1(n) = \tilde{f}_n(c) \]
\[ \Rightarrow \quad \tilde{f}_n(n) = n + \frac{1}{2} \]

Same connectedness argument
\[ \Rightarrow \quad \tilde{g}_0(n) = \tilde{g}_1(n) \]
\[ \Rightarrow \quad n + \frac{1}{2} = m + \frac{1}{2} \quad \Rightarrow \quad n = m. \]