The following theorem was supposed to have been proven in lecture 2. It wasn’t so now it becomes homework!

**Theorem 0.1 (Chuckrow, Jorgensen)** Let $\rho_n$ be a sequence of discrete faithful representations of a torsion free group $G$ in $\text{Isom}^+(\mathbb{H}^3)$ that converges to a representation $\rho$. If $G$ is not abelian then $\rho$ is discrete and faithful.

A discrete, faithful representation of $G$ is an injective homomorphism from $G$ to $\text{Isom}^+(\mathbb{H}^3)$ where the image is discrete. We say $\rho_n \to \rho$ if for all $g \in G$, $\rho_n(g) \to \rho(g)$ in $\text{Isom}(\mathbb{H}^3)$.

Here is one way to prove this.

1. If $\rho$ is not discrete show that (after possibly passing to subsequence) that there exists $g_n \in G \setminus \{\text{id}\}$ such that $\rho_n(g_n) \to \text{id}$.
2. Observe if $\rho$ is not faithful that there exists $g \in G \setminus \{\text{id}\}$ such that $\rho_n(g) \to \rho(g) = \text{id}$. In the remaining exercises we take $g_n = g$ to be a constant sequence when $\rho$ is not faithful.
3. Let $h \in G$ be an arbitrary element and show that $\rho_n([h, g_n]) \to \text{id}$.
4. Given any $p \in \mathbb{H}^3$ show that for large $n$ both $\rho_n(g_n)$ and $\rho_n([h, g_n])$ translate $p$ some distance $< \epsilon_3$ where $\epsilon_3$ is the 3-dimensional Margulis constant.
5. For large $n$ show that $[h, g_n]$ and $g_n$ commute.
6. If $a, b \in \text{Isom}^+(\mathbb{H}^3)$ show that if $a$ and $[a, b]$ commute then $a$ commutes with $b$. Use this to show that $h$ and $g_n$ commute for large $n$.
7. Show that if $a, b, c \in \text{Isom}^+(\mathbb{H}^3)$ and $a$ commutes with both $b$ and $c$ then $b$ commutes with $c$. Use this to show that $G$ is abelian. The proof is done!
8. We’ve actually shown something a bit stronger. Let $\Gamma_n = \rho_n(G)$ be the $\rho_n$-image of $G$. If $G$ is not abelian and $\rho_n$ converges then the identity is isolated in the union $\bigcup_n \Gamma_n$. Why?