1. Let $B$ be an $n \times n$-diagonal matrix with all diagonal entries 1 except the last one which is 
$-1$. Let $O(n - 1, 1)$ be the set of $n \times n$-matrices $A$ with $ABA^T = B$. Show that $O(n - 1, 1)$ 
is a sub-manifold of $R^{n^2}$. For $O(1,1)$ calculate the tangent space at the identity.

2. Let $U \subset R^n$ be a connected open set that with $p,q \in U$. Show that there exists a diffeomor-
phism $\phi$ of $R^n$ such that $\phi(p) = q$ and $\phi$ is the identity outside $U$. (Hint: First assume that
$U$ is convex with compact closure and find a vector field with support in $U$ whose flow takes
$p$ to $q$.)

3. Show that the tangent bundle of $S^2$ is not trivial.

4. Define $f : R^2 \rightarrow R^2$ by $f(x,y) = (x^2 - y^2, 2xy)$ and calculate $f^*dx$.

5. Let $X$ and $Y$ be manifolds and $f : X \rightarrow R^n$ and $g : Y \rightarrow R^n$ be smooth maps and let
$F : X \times Y \rightarrow R^n$ be defined by $F(x,y) = f(x) - g(y)$. Calculate the tangent map $F_*$ in terms
of $f_*$ and $g_*$. Make sure to justify your calculation. Let $f_a(x) = x + a$ for $a \in R^n$. Show that
for almost every $a \in R^n$, $f_a$ and $g$ are transverse.

6. Let $S^2 = \{(x,y,z)|x^2 + y^2 + z^2 = 1\}$ be the unit sphere and let $N \in \Lambda(R^3)$ be the vector
field on $R^3$ defined by $N(x,y,z) = (x,y,z)$. For any vectors $V,W \in T_pS^2$ the column vectors
$N(p),V$ and $W$ determine a matrix. Define a 2-form $\omega \in \Omega^2(S^2)$ by setting $\omega(p)(V,W)$ to be
the determinate of this matrix. Show that $S^2$ has an orientation such that for every oriented
basis $\{V,W\}$ of $T_pS^2$, $\omega(p)(V,W) > 0$. Use this to show that $\int_{S^2} \omega > 0$ and that $\omega$ is not
exact.