Review for Midterm 1 (Math 3210, Fall 2023)

1 Sample Problems

1. Use induction to prove that $n^2 < 3^n$ for all $n \in \mathbb{N}$.
2. Use induction to prove that $2^n < n!$ for all $n \geq 4$.
3. Use induction to show that
$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}.$$
4. Use induction to show that $7^{2n} - 1$ is divisible by 48 for every $n \in \mathbb{N}$.
5. Let F be a field. Let x, y, z be elements of F. Prove the following identities using only the axioms for fields, noting carefully where you are using each axiom at each step (if this problem were on the test, I would give you a list of the field axioms to look at). You may also use the identity $x \cdot 0 = 0$, which we proved in lecture.
   (a) Show that if $xz = yz$ and $z \neq 0$, then $x = y$.
   (b) Prove that if $xy = 0$ then either $x = 0$ or $y = 0$.
6. Prove that the following set is a Dedekind cut.
$$J = \left\{r \in \mathbb{Q} | r^3 < 5\right\}.$$
7. Prove that the Dedekind cut J from the previous problem is not equal to L_s for any rational s.
8. Let a and b be real numbers such that $a < b$. For each of the following sets, find the supremum and infimum.
   - $$A = [-\infty, b)$$
   - $$B = (-\infty, \infty)$$
   - $$C = [a, b)$$
   - $$D = \left\{\frac{n^2 - 1}{n^2 + 1} | n \in \mathbb{N}\right\}$$
   - $$E = \{r | r \in \mathbb{Q}, r > 0\}$$
   - $$F = \left\{r | r \in \mathbb{Q}, r^2 \leq 2\right\}.$$