1 Answers to sample problems

1. Proposition 13.3
2. Proposition 13.5
3. $m = 6$, $a = 2$, $b = 4$, $k = 3$.
4. $x = 14$
5. $5^{15} \cdot 2 = 14$.
6. We have $19 \cdot 7 - 6 \cdot 22 = 1$. So, the solution is $2^{19} \equiv 3 \pmod{23}$.
7. There are 2^7.
8. $\binom{7}{3}$.
9. $\binom{9}{1}(-2) = -18$.
10. $8\binom{20}{8}$.
11. $$k\binom{n}{k} = \frac{k \cdot n!}{k!(n-k)!} = \frac{n(n-1)!}{(k-1)!(n-k)!} = n\binom{n-1}{k-1}$$
12. Following the hint! There are $\binom{n}{k}$ ways to choose a team of k, and then k choices for who to be the captain. On the other hand, there are n choices for the captain, then $\binom{n-1}{k-1}$ choices for the rest of the team.
13. We follow the hint. We will show that the sum
$$\binom{k}{k} + \binom{k+1}{k} + \cdots + \binom{n}{k}$$
is equal to the number of subsets of $\{1, 2, \ldots, n, n+1\}$ which have size $k+1$. The number of subsets which have size $k+1$ and largest element j is zero if $j < k+1$. If $j \geq k+1$, then to specify a subset with largest element j, we have to select the remaining numbers in the subset. There are k of them, and they are all less than j. So the number of such subsets is $\binom{j-k-1}{k}$. Summing this over all j gives the answer.