Review for Midterm 1 (Math 2200, Spring 2023)

1 Sample Problems

1. Find an example of an infinite subset of $\mathbb{R}$ which is disjoint from $\mathbb{Q}$.
2. Find an example of two irrational numbers x and y such that xy is also irrational.
3. Prove or disprove the following statement: if x and y are irrational numbers, then xy is also irrational.
4. Using induction, prove that for any integer $n \geq 1$ we have
$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$$
5. Using induction, prove that for any integer $n \geq 1$ the number $10^{2n-1} + 1$ is divisible by 11.
6. Using induction, prove that, for every integer $n \geq 2$, we have
$$\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} \geq \frac{7}{12}$$
7. Prove that if n is an integer and $3|n^2$, then $3|n$.
8. Find an example of integers m and n such that $m|n^2$ but $m \nmid n$.
9. Using the Euclidean algorithm, find the highest common factor of 210 and 924.
10. Prove that $\sqrt[3]{5}$ is irrational. Write out a complete proof, explaining every step.
11. Prove that, if S is a set with a finite number of elements, then the numbers of subsets of S is equal to $2^{|S|}$ (here, $|S|$ denotes the number of elements of S).

2 Review topics by chapter

Chapter §1

- Understand set notation, including the subset ($S \subseteq T$) and elementhood ($x \in S$) relations.
- Understand the meaning of statements using quantifiers and set notation.
- Convert mathematical statements using sets and quantifiers into plain language, and vise versa.
- Be able to identify/write direct proofs and proofs by contradiction.

Chapter §2

- Know the definitions of the integers, the rational number, and the real numbers.

- Know the definition of an irrational number.
- Be able to prove that $\sqrt{2}$ is irrational.
- Understand the proof of Proposition 2.4.

Chapter §3

- Be familiar with the geometric series (Proposition 3.1)
- Know when it is the case that a number can have two different decimal expansions, and what the possibilities are (see Proposition 3.3).
- Know when a number has an eventually periodic decimal expansion (see Proposition 3.4).

Chapter §8

- Understand the statement of mathematical induction and strong mathematical induction (see "Principle of Mathematical Induction", "Principle of Mathematical Induction II", and "Principle of Strong Mathematical Induction").
- Know how to identify when a statement might be proved by induction, and when to use strong induction.
- Know how to write a proof using mathematical induction.

Chapter §10

- Know the definition of divisibility.
- Know the definition of the highest common factor of two integers.
- Understand how to apply the Euclidean algorithm to compute the highest common factor of two integers.
- Understand the statements of Propositions 10.1, 10.2, and 10.3, 10.5, and 10.6.
- Know the definition of a prime number and of coprime numbers.

Chapter §11

- Understand the statement of the fundamental theorem of arithmetic (Theorem 11.1).
- Understand the statement and proof of Proposition 11.3.