

$$\dot{x} = x(3-2x-y)$$

$$\dot{y} = y(2-x-y)$$

Nullclines: x -null: $x(3-2x-y)=0$

2 pieces: $\underline{x=0}$ and $\underline{y=3-2x}$

y -null: $y(2-x-y)=0$

2 pieces: $\underline{y=0}$ and $\underline{y=2-x}$

St. states:

$$\begin{cases} x(3-2x-y)=0 \\ y(2-x-y)=0 \end{cases}$$

$\begin{matrix} x=0 & \text{or} & y=3-2x \\ \downarrow & \text{plug in} & \downarrow \end{matrix}$

$$y(2-y)=0$$

$$y=0 \text{ or } y=2$$

$$y(2-x-3+2x)=0$$

$$y=0 \text{ or } x=1$$

(x^*, y^*) : $\boxed{(0,0) \quad (0,2)}$ & $\boxed{(\frac{3}{2},0) \quad (1,1)}$

Jacobian (linearization):

$$A = \begin{vmatrix} 3-4x-y & -x \\ -y & 2-x-2y \end{vmatrix}$$

$(0,0)$: $\begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix}$ $\lambda_1=3, \lambda_2=2 \Rightarrow (0,0)$ is unstable node

$(0,2)$: $\begin{vmatrix} 1 & 0 \\ -2 & -2 \end{vmatrix}$ $\lambda_1=1, \lambda_2=-2 \Rightarrow (0,2)$ is a saddle

$(\frac{3}{2},0)$: $\begin{vmatrix} -3 & -3/2 \\ 0 & 1/2 \end{vmatrix}$ $\lambda_1=-3, \lambda_2=1/2 \Rightarrow (\frac{3}{2},0)$ is a saddle

$(1,1)$: $\begin{vmatrix} -2 & -1 \\ -1 & -1 \end{vmatrix}$; $\begin{vmatrix} -2-\lambda & -1 \\ -1 & -1-\lambda \end{vmatrix} = \lambda^2 + 3\lambda + 2 - 1 = \lambda^2 + 3\lambda + 1 = 0$
 $\lambda = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}$; stable node

