4.5.1 Consider the alternative firefly model

(1) $\dot{\Theta} = \Omega$

(2) $\dot{\theta} = \omega + A f(\Theta - \theta)$, where

$$f(\varphi) = \begin{cases} \varphi, & -\frac{\pi}{2} < \varphi < \frac{\pi}{2} \\ \pi - \varphi, & \frac{\pi}{2} < \varphi < \frac{3\pi}{2} \end{cases}$$ and extended periodically.

Find the range of entrainment.

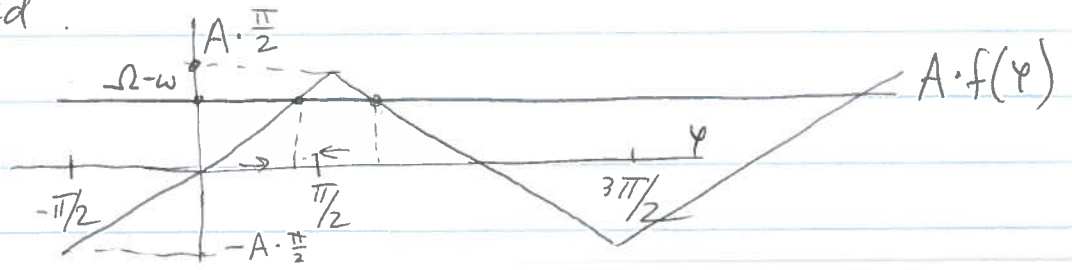
Solution: the range of entrainment is the range of the stimulus frequencies (the range of Ω) that allow the firefly to phase lock to it (i.e. "be entrained" by the stimulus).

Phase locked solution means that $\varphi = \Theta - \theta = \text{const}$ is a solution, or, in other words, that the equation for φ has a stable fixed point solution.

Equation for (φ): $\dot{\varphi} = \dot{\Theta} - \dot{\theta} = (\Omega - \omega) - A f(\Theta - \theta)$

$$\dot{\varphi} = (\Omega - \omega) - A f(\varphi).$$

We need to study fixed points as parameter Ω is varied.



Fixed points equation:

$$\Omega - \omega = A f(\varphi^*).$$

From the graph above, it has a solution as long as

$$|\Omega - \omega| \leq A \cdot \frac{\pi}{2}$$

and one of the fixed points will be stable

$$-A \cdot \frac{\pi}{2} \leq \Omega - \omega \leq A \cdot \frac{\pi}{2}$$

$$\omega - A \cdot \frac{\pi}{2} \leq \Omega \leq \omega + A \frac{\pi}{2}$$

$\left(\omega - A\frac{\pi}{2}, \omega + A\frac{\pi}{2}\right)$ is the range of entrainment.

Note: I tried to write a detailed solution, but only the pieces with a vertical bar next to them would be sufficient for a complete quiz solution.