Week 2 exercises

1. (I. Kapovich-Rafi) Let $f : X \to Y$ be a Lipschitz map between two graphs that sends vertices to vertices and is surjective on the vertices. Suppose there is some D such that whenever $[x, x']$ is a geodesic in X and $d(f(x), f(x')) \le 1$ then $diam f([x, x']) \le D$. If X is hyperbolic, show that Y is also hyperbolic.

2. Let a, b be two curves in Σ that intersect once (for concreteness).

   (a) For large n consider $T_a^n(b)$. Show that $d_A(b, T_a^n(b))$ is large (on the order of n), where A is the annulus around a.

   (b) If Y is any subsurface such that the projections of $a, b, T_a^n(b)$ are all defined, then $d_Y(b, T_a^n(b))$ is bounded independently of n. (In fact, $Y = A$ is the only subsurface with large projection distance but to prove this you would need the following fact: if $Y \subset Z$ are two subsurfaces then $\pi_Y(\gamma) = \pi_Y \pi_Z(\gamma)$; this is clear when Y is not an annulus, but the annulus case requires a proof.)

   (c) Show that $d_B(a, T_b^n T_a(b))$ is large for the annulus B around b. There is one more annulus C with $d_C(a, T_b^n T_a(b))$ large. Find it. Hint: $d_{f(Y)}(f(\alpha), f(\beta)) = d_Y(\alpha, \beta)$.