Math 4800/6080. Week One Starter Problem

Consider the polynomial equation:

$$x^2 - 2y^2 = 1$$

What can you say about:

(1) The real solutions to this equation (as a subset of the plane $\mathbb{R}^2$)?

(2) The rational solutions
(i.e. real solutions both of whose coordinates are rational numbers)?

(3) The integer solutions?

(4) The complex solutions?

(5) Solutions consisting of pairs of elements of a finite field $\mathbb{F}_p$
(p is a prime)?