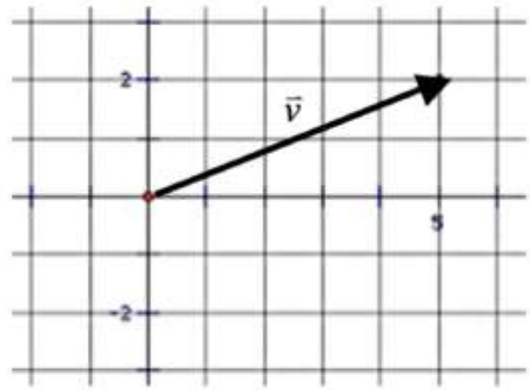
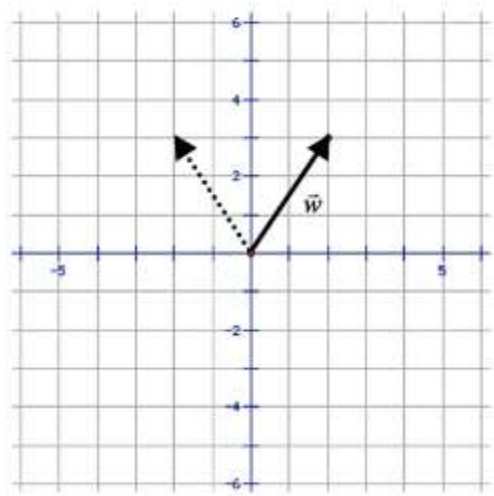
Transformation with Matrices

Various notations are used to denote vectors: bold-faced type, ***v***; a variable written with a harpoon over it, $\vec{v}$; or listing the horizontal and vertical components of the vector, $\langle v_x . v_y \rangle$. In this task we will represent vectors by listing their horizontal and vertical components in a matrix with a single column, $\begin{bmatrix} v_x \\ v_y \end{bmatrix}$.



1. Represent the vector labeled $\vec{v}$ in the diagram at the right as a matrix with one column.



Matrix multiplication can be used to transform vectors and images in a plane.

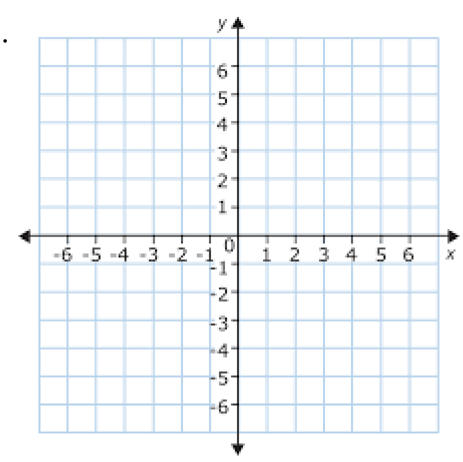
Suppose we want to reflect $\vec{w}$ over the *y*-axis. We can represent $\vec{w}$ with the matrix $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and the reflected vector with the matrix $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

2. Find the 2 × 2 matrix that we can multiply the matrix representing the original vector by in order to obtain the matrix that represent the reflected vector. That is, find *a*, *b*, *c*, and *d* such that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

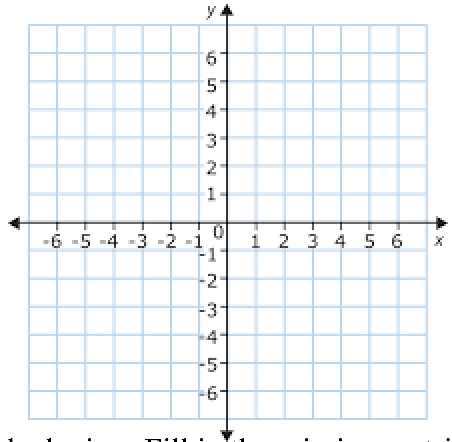
3. Reflect the original vector over the y-axis. Fill in the missing matrix with the reflecte vector. Then find a, b, c and d for the matrix that will reflect $\vec{w}$ over the *x*-axis.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$



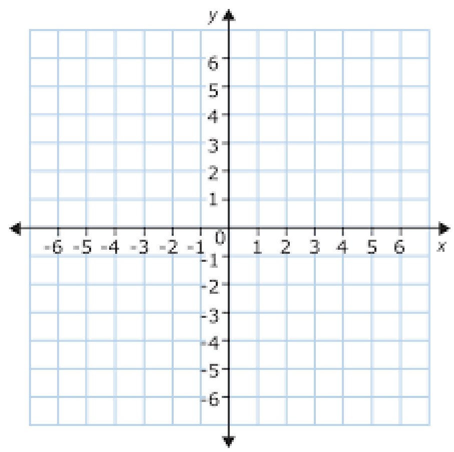
4. Rotate the original vector, $\vec{w}$, 90° counterclockwise. Fill in the missing matrix with the reflecte vector. Then find a, b, c and d for the matrix that will rotate $\vec{w}$ 90° counterclockwise about the origin.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$



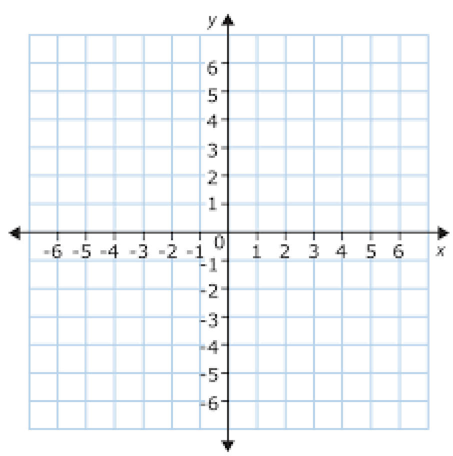
5. Rotate the original vector, $\vec{w}$, 180° counterclockwise. Fill in the missing matrix with the reflecte vector. Then find a, b, c and d for the matrix that will rotate $\vec{w}$ 180° counterclockwise about the origin.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

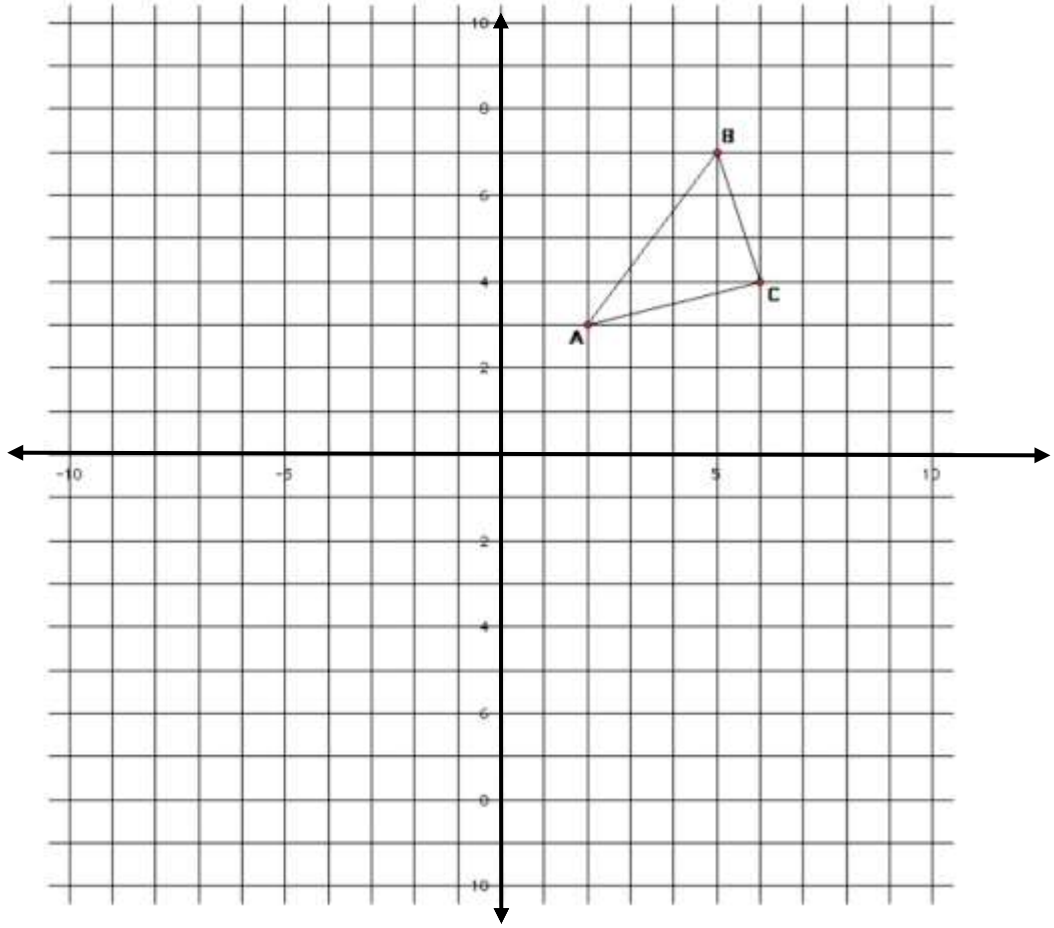


6. Rotate the original vector, $\vec{w}$, 270° counterclockwise. Fill in the missing matrix with the reflecte vector. Then find a, b, c and d for the matrix that will rotate $\vec{w}$ 270° counterclockwise about the origin.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$



We can represent polygons in the plane by listing the coordinates of its vertices as columns of a matrix. For example, the triangle below can be represented by the matrix $\begin{bmatrix} 2 & 5 & 6 \\ 3 & 7 & 4 \end{bmatrix}$.



8. Multiply this matrix, which represents the vertices of ΔABC, by the matrix found in question 2. Interpret the product matrix as representing the coordinates of the vertices of another triangle in the plane. Plot these points and sketch the triangle on the graph above. How is this new triangle related to the original triangle?

9. How might you find the coordinates of the triangle that is formed after ΔABC, is rotated 90° counterclockwise about the origin using matrix multiplication. Find the coordinates of the rotated triangle.

10. How might you find the coordinates of the triangle that is formed after ΔABC is reflected over the x-axis using matrix multiplication? Find the coordinates of the reflected triangle.