Math 4800/6080 Project Ideas
Polynomials (Spring 2013)

1. Diophantine Equations. This is the problem of finding *integer solutions* to a polynomial equation (or system of polynomial equations) with *integer* coefficients. Examples you might explore include:

(a) Fermat's Last Theorem. No integer solutions to $x^n + y^n = z^n$ other than the obvious ones when $n \geq 3$ [1].
Obviously not the general case, but some history and the cases:

$$x^4 + y^4 = z^4$$
$$x^3 + y^3 = z^3$$

(b) Pell's Equation. How to find all solutions to:

$$x^2 - dy^2 = \pm 1$$

when d is a positive integer which is not a perfect square [2].

(c) Integer solutions to elliptic curve equations:

$$y^2 = x^3 + ax^2 + bx + c$$

especially the Nagell-Lutz Theorem [3].

2. More on the Projective Plane.

There is a **lot** more material than we have covered e.g. [4].

Choose a topic that interests you and clear it with me.

3. Conics.

There is also a lot more material on conics, e.g. [5].

4. Hilbert's 16th Problem (First Part). [6].

Give some history and discuss Harnack's inequality and M-curves.

5. Elliptic Curves.

(a) The Nagell-Lutz theorem on torsion subgroups (see above).

(b) Mordell's Theorem on finite-generators for rational solutions [3].

(c) Verification of the list of finite torsion subgroups.

(d) Applications of elliptic curves to cryptography.

[1] Wiki: Proof of Fermat's Last Theorem for specific exponents.
The page has lots of good book references.

[2] Again, there is a good Wiki: Pell's Equation (with good references).
A classic on this is Davenport, The Higher Arithmetic.

[3] See Silverman and Tate, Rational Points on Elliptic Curves.

[4] See Coxeter (or any other author), Projective Geometry.

[5] Kendig, Conics

[6] http://www.math.sunysb.edu/~oleg/introMSRI.pdf
Easy reading on the topology or real plane algebraic curves.