Math 4030-001/Foundations of Algebra/Fall 2017
First Midterm

1. (30 points) Convert each of the following English sentences into mathematical statements with appropriate quantifiers:

(a) The cube of an integer is odd if and only if the integer is odd.

(b) The product of two non-zero integers cannot be zero.

(c) There is a pair of non-zero integers whose sum is zero.

(d) There is neither a largest nor a smallest integer.

(e) Every set of integers with a lower bound is either the empty set or else it has a unique smallest element.

(f) There is no rational number whose square is 27.

2. (30 points) Prove all of the six statements in # 1.

3. (10 points) Prove the following statement for all n by induction:

$$(1 + \cdots + x^n)(1 - x) = 1 - x^{n+1}$$

4. (10 points) Show that 1007 and 493 are relatively prime and solve:

$$a(1007) + b(493) = 1$$

with integers a and b. Show all your work!

5. (10 points) Write a proof of unique factorization of natural numbers in your own words. (First prove factorization and then uniqueness.)

6. (10 points) Suppose n is a natural number and there is a rational number r so that $r^2 = n$. Show that r must be an integer.