Name: Key

Math 1310-004
Quiz 9
November 6, 2014

1. (5 points) Find the following limit:

$$\lim_{x\to 0}(1+2x)^{\frac{1}{x}}$$

$\lim_{x\to 0} e^{\frac{1}{x}\ln(1+2x)} = e^2$

$\lim_{x\to 0} \frac{\ln(1+2x)}{x} \overset{(l'Hôpital)}{=} \lim_{x\to 0} \frac{\frac{1}{1+2x}\cdot 2}{1} = 2$ (Chain Rule)

2. (5 points) You are asked to construct a cylinder with a fixed volume V and minimal surface area. What is the ratio of the height of your cylinder to its radius? ($V = \pi r^2 h, \quad SA = 2\pi r^2 + 2\pi rh$)

$V = \pi r^2 h \; ; \quad h = \frac{V}{\pi r^2}$

$SA = f(r) = 2\pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right) = 2\pi r^2 + \frac{2V}{r}$

$f'(r) = 4\pi r - \frac{2V}{r^2} \; ; \; f'(r)=0 \Leftrightarrow 4\pi r = \frac{2V}{r^2}$

$r = \sqrt[3]{\frac{V}{2\pi}} = V^{1/3}2^{-1/3}\pi^{-1/3} \; ; \quad h = V\cdot\pi^{-1}r^{-2}$

$= V\cdot\pi^{-1}\cdot V^{-2/3}\cdot 2^{2/3}\cdot\pi^{+2/3}$

$= V^{1/3}\cdot 2^{2/3}\cdot\pi^{-1/3} = r\cdot 2$

$\boxed{h = 2r}$