Math 1310-004

First Midterm Sample

1. Given the graph of the function $y = f(x)$ below:

(a) What are the Domain and Range of $f(x)$?

(b) For what values of a does $\lim_{x\to a} f(x)$ exist?

(c) For what values of a is $f(x)$ continuous at a?

(d) For what values of a does $f'(a)$ exist?

(e) For what values of a is $f'(a)$ positive? negative? zero?

(f) For what values of a is $f''(a)$ positive? negative? zero?

2. Using the following table:

x	f(x)
-0.2	-0.16
-0.1	-0.09
0	0
0.1	0.11
0.2	0.24

(i) Estimate the value of $f'(0)$.

(ii) Estimate the value of $f''(0)$.

3. Find the following limits algebraically. Show your work.

(a)

$$\lim_{x\to 1} \frac{x^2 - 1}{x^3 - 1}$$

(b)

$$\lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$

4. The hyperbolic tangent is the function:

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Find the derivative $\tanh'(x)$ and show all your work.

5. Find the domain, range and all critical points of the function:

$$f(x) = \frac{x^2 + 2x - 1}{x + 1}$$

Explain how you got your answers.

6. Compute the derivatives of the following functions:

(a)

$$\sin^3(x)$$

(b)

$$\sin(x^3)$$

(c)

$$\sin(\sin(\sin(x)))$$

Extra Credit. Prove the reciprocal rule for derivatives:

$$(1/g)' = -\frac{g'}{g^2}$$

and use it, together with the product rule, to prove the quotient rule.

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The Cheat Sheet

Rules for Differentiating Combinations of Functions.

$$(cf)' = cf'$$

$$(f+g)' = f' + g', \quad (f-g)' = f' - g'$$

$$(fg)' = f'g + fg', \quad (1/g)' = -\frac{g'}{g^2}, \quad (f/g)' = \frac{f'g - fg'}{g^2}$$

Let $u = g(x)$ and $F(x) = f(g(x)) = f(u)$. Then $F'(x) = f'(u) \cdot g'(x)$.

The Basic Derivatives (so far).

$$\frac{d}{dx}(c) = 0, \quad \frac{d}{dx}(x^n) = nx^{n-1} \text{ as long as } n \neq 0$$

$$\frac{d}{dx}(e^x) = e^x, \quad \frac{d}{dx}(c^x) = c^x \ln(c) \text{ for all } c > 0$$

$$\frac{d}{dx}(\sin(x)) = \cos(x), \quad \frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x), \quad \frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x), \quad \frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$$

Some Trig Identities.

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

Exponent and Log Rules. Assume $b, c > 0$

$$c^{x+y} = c^x c^y, \quad \log_c(xy) = \log_c(x) + \log_c(y)$$

$$c^{-x} = 1/c^x, \quad \log_c(1/x) = -\log_c(x)$$

$$(c^x)^y = c^{xy}, \quad \log_c(x^y) = y \log_c(x)$$

$$b^x c^x = (bc)^x, \quad \log_b(c) = \ln(c)/\ln(b)$$

e is the number (approximate value: 2.718) satisfying:

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

and $\ln(x) = \log_e(x)$ is the "natural log."