Name:_____

Math 1310-004/Bertram
Second Midterm Examination
November 10, 2014

Please indicate your reasoning and show all your work. You may use a graphing calculator, but an unsupported answer will not get credit.

Relax and good luck!

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20 points) Find the derivative of the function:

$$f(x) = \sqrt{\frac{e^{x^2}}{x^2+1}}$$

using logarithmic differentiation (do not simplify your answer).

2. (20 points) Find a linear approximation for the function:

$$f(x) = \sqrt[4]{x} \text{ near } x = 16$$

and use it to get an approximate value for $\sqrt[4]{18}$.

3. Compute the following limits.

(a) (5 points)

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} =$$

(b) (5 points)

$$\lim_{x\to 0} x \ln(x) =$$

(c) (5 points)

$$\lim_{x\to \infty} x \ln(x) =$$

(d) (5 points)

$$\lim_{x\to 0} (1 + 3x + x^2)^{1/x} =$$

4. (20 points) Find the distance between the point $(1, 4)$ and the closest point to it on the line:

$$y = 2x + 1$$

5. (20 points) Boat A is 10 km West of Boat B at Noon. If Boat A moves North at a constant speed of 10 km/hr and Boat B moves South at a constant speed of 20 km/hr, then how fast are the boats moving apart from one another at 3PM?

Note. Pay attention to the directions in which the boats are moving. They are not the same as in the homework!

Extra Credit. (10 points) You are to build a cone (with no base) of a fixed volume V with the minimum amount of material. What is the ratio of height to radius h/r of your cone?

Useful Formulas.

- Volume of a cone: $\frac{1}{3}\pi r^2 h$.
- Surface area of the cone (without the base): $\pi r\sqrt{r^2+h^2}$.

Math 1310-004 The Official Cheat Sheet

Rules for Differentiating Combinations of Functions.

$$(cf)' = cf', \quad (f+g)' = f' + g', \quad (f-g)' = f' - g'$$

$$(fg)' = f'g + fg', \quad (f/g)' = \frac{f'g - fg'}{g^2}, \quad \frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$$

The Basic Derivatives.

$$\frac{d}{dx}(c) = 0, \quad \frac{d}{dx}(x^n) = nx^{n-1} \text{ if } n \neq 0$$

$$\frac{d}{dx}(e^x) = e^x, \quad \frac{d}{dx}(c^x) = c^x \ln(c), \quad \frac{d}{dx}\ln(x) = \frac{1}{x}, \quad \frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}(\sin(x)) = \cos(x), \quad \frac{d}{dx}(\cos(x)) = -\sin(x), \quad \frac{d}{dx}(\tan(x)) = \sec^2(x)$$

$$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x), \quad \frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x), \quad \frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

Some Trig Identities. $\sin^2(x) + \cos^2(x) = 1$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$

$$\sin(2x) = 2\sin(x)\cos(x), \ \cos(2x) = \sin^2(x) - \cos^2(x)$$

Exponent and Log Rules.

$$c^{x+y} = c^x c^y, \quad c^{x-y} = c^x/c^y, \quad (c^x)^y = c^{xy}, \quad b^x c^x = (bc)^x$$

$$\log(xy) = \log(x) + \log(y), \quad \log(x^y) = y\log(x)$$

$$\log(x/y) = \log(x) - \log(y), \quad \log_b(x) = \ln(x)/\ln(b)$$

Miscellaneous Stuff:

$f(x)^{g(x)} = e^{g(x)\ln(f(x))}$ may be used to apply l'Hôpital's rule.

$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}$ is used for logarithmic differentiation.

Linear Approximations: Given $f(a)$ and $f'(a)$, then:

$$L_a(b) = f(a) + f'(a)(b-a)$$

is the linear approximation of $f(b)$ for b "near" a.