

Although it is possible to eliminate the parameter  $\theta$  from Equations 1, the resulting Cartesian equation in  $x$  and  $y$  is very complicated and not as convenient to work with as the parametric equations.

One of the first people to study the cycloid was Galileo, who proposed that bridges be built in the shape of cycloids and who tried to find the area under one arch of a cycloid. Later this curve arose in connection with the **brachistochrone problem**: Find the curve along which a particle will slide in the shortest time (under the influence of gravity) from a point  $A$  to a lower point  $B$  not directly beneath  $A$ . The Swiss mathematician John Bernoulli, who posed this problem in 1696, showed that among all possible curves that join  $A$  to  $B$ , as in Figure 15, the particle will take the least time sliding from  $A$  to  $B$  if the curve is part of an inverted arch of a cycloid.

The Dutch physicist Huygens had already shown that the cycloid is also the solution to the **tautochrone problem**; that is, no matter where a particle  $P$  is placed on an inverted cycloid, it takes the same time to slide to the bottom (see Figure 16). Huygens proposed that pendulum clocks (which he invented) should swing in cycloidal arcs because then the pendulum would take the same time to make a complete oscillation whether it swings through a wide or a small arc.



ing the parametric equations to plot w the direction in which the curve is

$$\begin{aligned} & , \quad -2 \leq t \leq 2 \\ & -3 \leq t \leq 3 \\ & 1t, \quad 0 \leq t \leq \pi/2 \\ & t, \quad -2 \leq t \leq 2 \end{aligned}$$

g the parametric equations to plot arrow the direction in which the curve to find a Cartesian equation of the

$$\begin{aligned} & 1 \\ & t^2 \end{aligned}$$

to find a Cartesian equation of the licate with an arrow the direction in 1 as the parameter increases.

$$-\pi \leq \theta \leq \pi$$

puter with graphing software required

1. Homework Hints available in TEC

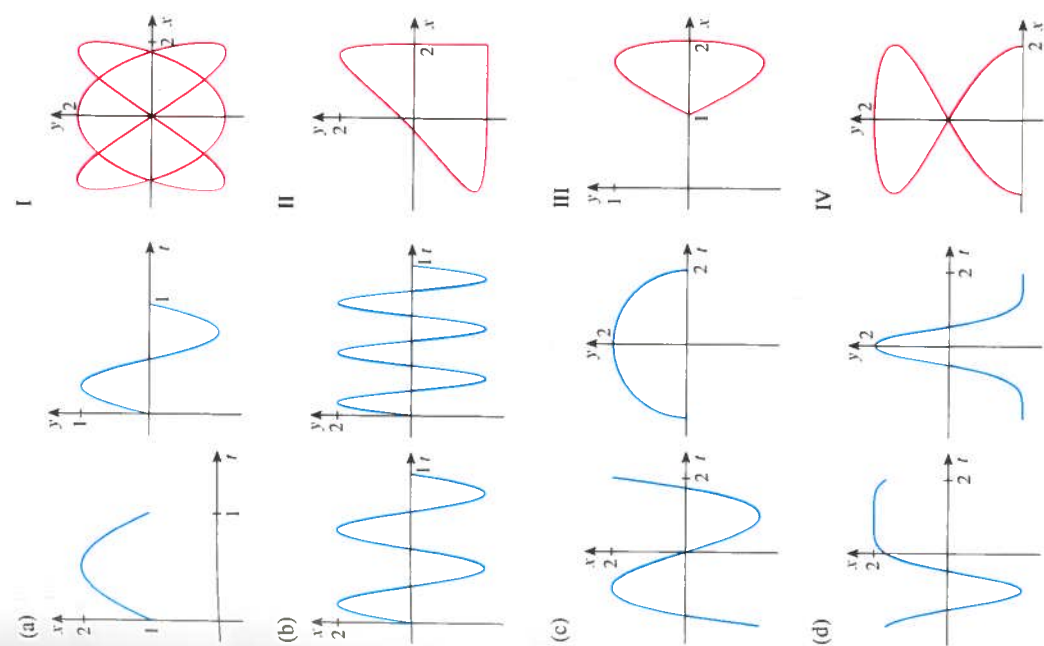
10.  $x = \frac{1}{2} \cos \theta$ ,  $y = 2 \sin \theta$ ,  $0 \leq \theta \leq \pi$
11.  $x = \sin t$ ,  $y = \csc t$ ,  $0 < t < \pi/2$
12.  $x = \tan^2 \theta$ ,  $y = \sec \theta$ ,  $-\pi/2 < \theta < \pi/2$
13.  $x = e^{2t}$ ,  $y = t + 1$
14.  $x = e^t - 1$ ,  $y = e^{2t}$
15.  $x = \sin \theta$ ,  $y = \cos 2\theta$
16.  $x = \ln t$ ,  $y = \sqrt{t}$ ,  $t \geq 1$

17–20 Describe the motion of a particle with position  $(x, y)$  as  $t$  varies in the given interval.

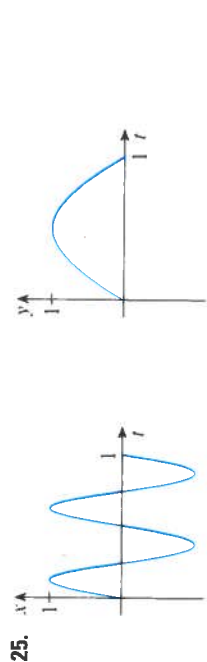
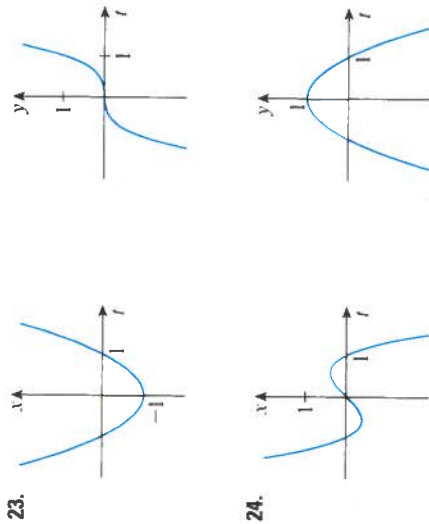
17.  $x = 3 + 2 \cos t$ ,  $y = 1 + 2 \sin t$ ,  $\pi/2 \leq t \leq 3\pi/2$
18.  $x = 2 \sin t$ ,  $y = 4 + \cos t$ ,  $0 \leq t \leq 3\pi/2$
19.  $x = 5 \sin t$ ,  $y = 2 \cos t$ ,  $-\pi \leq t \leq 5\pi$
20.  $x = \sin t$ ,  $y = \cos^2 t$ ,  $-2\pi \leq t \leq 2\pi$

21. Suppose a curve is given by the parametric equations  $x = f(t)$ ,  $y = g(t)$ , where the range of  $f$  is  $[1, 4]$  and the range of  $g$  is  $[2, 3]$ . What can you say about the curve?

22. Match the graphs of the parametric equations  $x = f(t)$  and  $y = g(t)$  in (a)–(d) with the parametric curves labeled I–IV. Give reasons for your choices.

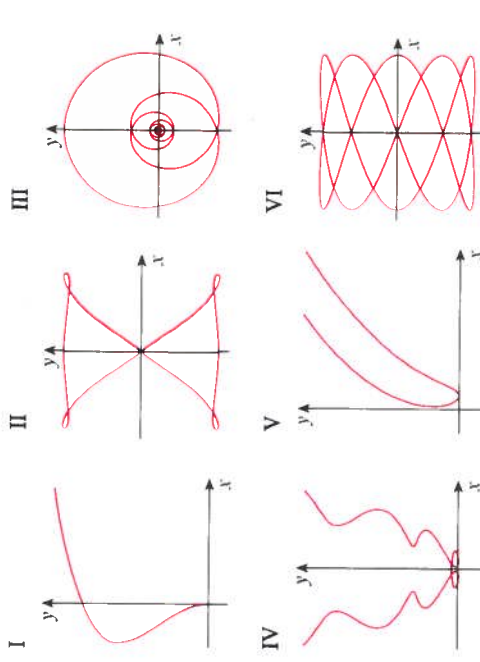


23–25 Use the graphs of  $x = f(t)$  and  $y = g(t)$  to sketch the parametric curve  $x = f(t)$ ,  $y = g(t)$ . Indicate with arrows the direction in which the curve is traced as  $t$  increases.



26. Match the parametric equations with the graphs labeled I–VI. Give reasons for your choices. (Do not use a graphing device.)

- (a)  $x = t^4 - t + 1$ ,  $y = t^2$
- (b)  $x = t^2 - 2t$ ,  $y = \sqrt{t}$
- (c)  $x = \sin 2t$ ,  $y = \sin(t + \sin 2t)$
- (d)  $x = \cos 5t$ ,  $y = \sin 2t$
- (e)  $x = t + \sin 4t$ ,  $y = t^2 + \cos 3t$
- (f)  $x = \frac{\sin 2t}{4 + t^2}$ ,  $y = \frac{\cos 2t}{4 + t^2}$



27. Graph the curve  $x = y - 2 \sin \pi y$ .

28. Graph the curves  $y = x^3 - 4x$  and  $x = y^3 - 4y$  and find their points of intersection correct to one decimal place.

29. (a) Show that the parametric equations

$$x = x_1 + (x_2 - x_1)t \quad y = y_1 + (y_2 - y_1)t$$

where  $0 \leq t \leq 1$ , describe the line segment that joins the points  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$ .

(b) Find parametric equations to represent the line segment from  $(-2, 7)$  to  $(3, -1)$ .

30. Use a graphing device and the result of Exercise 29(a) to draw the triangle with vertices  $A(1, 1)$ ,  $B(4, 2)$ , and  $C(1, 5)$ .

31. Find parametric equations for the path of a particle that moves along the circle  $x^2 + (y - 1)^2 = 4$  in the manner described.

- (a) Once around clockwise, starting at  $(2, 1)$
- (b) Three times around counterclockwise, starting at  $(2, 1)$
- (c) Halfway around counterclockwise, starting at  $(0, 3)$