...osition (or *composite*) of $f$ and $g$ and is denoted by $f \circ g$

...ns $f$ and $g$, the **composite function** $f \circ g$ (also called ...) is defined by

$$(f \circ g)(x) = f(g(x))$$

...t of all $x$ in the domain of $g$ such that $g(x)$ is in the domain ...s defined whenever both $g(x)$ and $f(g(x))$ are defined. Fig- ... $g$ in terms of machines.

... If $f(x) = x^2$ and $g(x) = x - 3$, find the composite

$$= f(g(x)) = f(x - 3) = (x - 3)^2$$

$$= g(f(x)) = g(x^2) = x^2 - 3$$

...ample 6 that, in general, $f \circ g \neq g \circ f$. Remember, the ...nction $g$ is applied first and then $f$ is applied second. In ...hat *first* subtracts 3 and *then* squares; $g \circ f$ is the function ...ts 3.

...d $g(x) = \sqrt{2 - x}$, find each function and its domain.
...) $f \circ f$ (d) $g \circ g$

$$) = f(\sqrt{2 - x}) = \sqrt{\sqrt{2 - x}} = \sqrt[4]{2 - x}$$

$$\geq 0\} = \{x \mid x \leq 2\} = (-\infty, 2].$$

$$= g(f(x)) = g(\sqrt{x}) = \sqrt{2 - \sqrt{x}}$$

...e $x \geq 0$. For $\sqrt{2 - \sqrt{x}}$ to be defined we must have ...$x \leq 4$. Thus we have $0 \leq x \leq 4$, so the domain of

$$f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$$

$$(f \circ g \circ h)(x) = f(g(h(x)))$$

**EXAMPLE 8** Find $f \circ g \circ h$ if $f(x) = x/(x + 1)$, $g(x) = x^{10}$, and $h(x) = x + 3$.

SOLUTION

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x + 3))$$

$$= f((x + 3)^{10}) = \frac{(x + 3)^{10}}{(x + 3)^{10} + 1}$$

So far we have used composition to build complicated functions from simpler ones. But in calculus it is often useful to be able to *decompose* a complicated function into simpler ones, as in the following example.

**EXAMPLE 9** **Decomposing a function** Given $F(x) = \cos^2(x + 9)$, find functions $f$, $g$, and $h$ such that $F = f \circ g \circ h$.

SOLUTION Since $F(x) = [\cos(x + 9)]^2$, the formula for $F$ says: First add 9, then take the cosine of the result, and finally square. So we let

$$h(x) = x + 9 \qquad g(x) = \cos x \qquad f(x) = x^2$$

Then

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x + 9)) = f(\cos(x + 9))$$

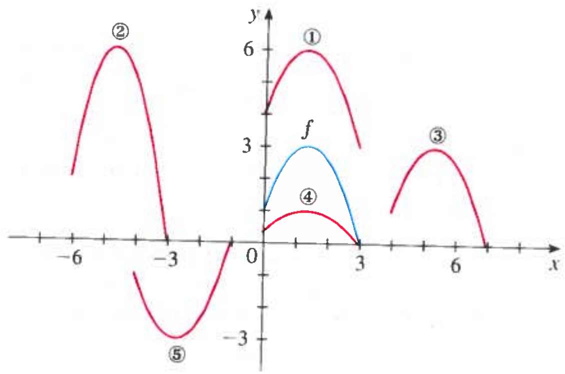
$$= [\cos(x + 9)]^2 = F(x)$$

## 1.3 Exercises

1. Suppose the graph of $f$ is given. Write equations for the graphs that are obtained from the graph of $f$ as follows.
   (a) Shift 3 units upward.
   (b) Shift 3 units downward.
   (c) Shift 3 units to the right.
   (d) Shift 3 units to the left.
   (e) Reflect about the $x$-axis.
   (f) Reflect about the $y$-axis.
   (g) Stretch vertically by a factor of 3.
   (h) Shrink vertically by a factor of 3.

2. Explain how each graph is obtained from the graph of $y = f(x)$.
   (a) $y = f(x) + 8$ (b) $y = f(x + 8)$
   (c) $y = 8f(x)$ (d) $y = f(8x)$
   (e) $y = -f(x) - 1$ (f) $y = 8f(\frac{1}{8}x)$

3. The graph of $y = f(x)$ is given. Match each equation with its graph and give reasons for your choices.
   (a) $y = f(x - 4)$ (b) $y = f(x) + 3$
   (c) $y = \frac{1}{3}f(x)$ (d) $y = -f(x + 4)$
   (e) $y = 2f(x + 6)$




1. Homework Hints available in TEC