

Math 1310-004
Sample Final Exam

1. In the space provided, sketch the graph of a function $f(x)$ satisfying:
- (a) The domain of $f(x)$ is $(-\infty, \infty)$ and the range of $f(x)$ is $[-1, 1]$.
 - (b) $f(x)$ is continuous and differentiable everywhere **except** at $x = 0$, where it is not continuous.
 - (c) $f(-1) = 0$, $f'(-1) = 0$ and $f''(-1) > 0$.
 - (d) $f(0) = 1$.
 - (e) $f(1) = 0$, $f'(1) = 0$ and $f''(1) < 0$.
 - (f) $\lim_{x \rightarrow -\infty} f(x) = -1$ and $\lim_{x \rightarrow \infty} f(x) = 1$.
2. Find the first and second derivatives of the function $f(x) = x \tan^{-1}(x)$.
3. Find the points on the parabola $y = x^2$ that are closest to $(0, a)$:
- (a) In the case $a < \frac{1}{2}$.
 - (b) In the case $a > \frac{1}{2}$.
- (The answers to (a) and (b) are different!)
4. Using the following table:

x	$f(x)$
0	1.0
0.2	1.08
0.4	1.64
0.6	3.16
0.8	6.12

- (a) Estimate the value of $f'(0.5)$.
 - (b) Estimate the value of $\int_0^{0.8} f(x)dx$ using the trapezoid rule.
 - (c) Estimate the value of $\int_0^{0.8} f(x)dx$ using Simpson's rule.
5. Find the following indefinite integrals. Show your work!
- (a)

$$\int \ln(x)dx =$$

- (b)

$$\int \tan^{-1}(x)dx =$$

6. Find the following definite integrals. Show your work!

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(a)

$$\int_0^1 \sqrt{1-x^2} dx =$$

(b)

$$\int_0^1 x\sqrt{1-x^2} dx =$$

7. Use the fact that:

$$\frac{1}{1-x^2} = \frac{1}{2} \left(\frac{1}{1-x} - \frac{1}{1+x} \right)$$

help you to compute (some of) the following integrals:

(a)

$$\int \frac{1}{1-x^2} dx =$$

(b)

$$\int \frac{x}{1-x^2} dx =$$

(c)

$$\int \frac{x^2}{1-x^2} dx =$$

8. Compute the following indefinite integral by parts:

$$\int \cos^2(x) dx =$$

Recompute the same integral using the trig identity:

$$\cos(2x) = 2\cos^2(x) - 1$$

What does this say about $\sin(2x)$?

Remark. There may be a problem on the final in which the integrals are improper.