The Hyperbola

Math 1220 (Spring 2003)

The standard equation for a hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ with } a > b > 0$$

Once that is given to us, we define a focus and directrix:

$$c = \sqrt{a^2 + b^2}$$

$$F = (c, 0)$$

$$e = \frac{c}{a} > 1$$

$$L = \{x = \frac{a}{e}\}$$

and the new quantity e is called the **eccentricity** of the hyperbola.

Remark on Eccentricity: The eccentricity is the value e so that:

$$|PF| = e|PL|$$

is the equation of the conic (parabola, ellipse or hyperbola). Thus $e = 1$ for a parabola, $e < 1$ for an ellipse, and as we'll see $e > 1$ for a hyperbola. What about $e = 0$? In the case of an ellipse, the eccentricity measures its "flatness" so that an e close to 1 looks more like a hotdog, and an e close to 0 looks more like a circle. Similarly, for hyperbolas, an e close to 1 looks like two parabolas, but an e very large (close to ∞) looks very flat.

How to Draw the Hyperbola: The values $\frac{b}{a}$ and $-\frac{b}{a}$ are the slopes of the asymptotes to the hyperbola. Thus, to sketch it, you need to draw these asymptotes (crossing at the origin) and remember that the hyperbola passes through the points $(a, 0)$ and $(-a, 0)$. Then sketch!

To Show: $|PF| = e|PL|$ for the hyperbola given above.

Same as the ellipse proof.

String Property: If $F' = (-c, 0)$ and P is on the hyperbola, then:

$$|PF'| - |PF| = 2a$$

if P is closer to F. If P is closer to F' (i.e. on the left) then:

$$|PF| - |PF'| = 2a$$

Optical Property: If light the outside of a hyperbola is a mirror, and a light beam is aimed towards F (from outside) then it will bounce to point towards F', and vice versa.

Both these properties are absolutely analogous to the properties of the ellipse, and are proved the same way.

Remark: A parabola and a hyperbola together will make a good telescope. Why?