Name:________________________________

Math 1040

Midterm Examination

April 15, 2016

Relax and good luck!

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20 points) 100 households were surveyed and the number of households owning x cars was tabulated in two columns of the table below.

Fill out the rest of the table and find the expected number of cars owned and the standard deviation and put your answers in the spaces provided.

x	Households	$P(x)$	$x-\mu$	$(x-\mu)^2P(x)$
0	10	.1	−1.7	0.3
1	30	.3	−.7	0.15
2	45	.45	.3	0.05
3	10	.1	1.3	0.2
4	5	.05	2.3	0.3

$0\times .1 + 1\times .2 + 2\times .45 + 3\times .1 + 4\times .05 =$

Expected Value $= \mu = \sum xP(x) =$ 1.7

Standard Deviation $= \sigma = \sqrt{\sum(x-\mu)^2P(x)} =$ $\sqrt{1} \approx 1$

(more accurate: $\sqrt{.9} \approx .95$)

2. You will need the three formulas below about the discrete probability distribution coming from a *binomial experiment* with:

$$n = \text{the number of trials}$$
$$p = \text{probability of success}$$
$$q = \text{probability of failure}$$

The formulas you will need are:

$$P(x) = {}_nC_x p^x q^{n-x}, \qquad \mu = np, \qquad \sigma = \sqrt{npq}$$

So here is the problem:

A celebrated doctor has a 90% success rate with a medical procedure. He performs the procedure on 5 patients.

(a) (5 points) Find n, p and q.

n = 5, **p** = .9, **q** = .1

(b) (5 points) What is the probability that all 5 procedures are successes?
(Hint: ${}_5C_5 = 1$)

$1 \times (.9)^5 \times (.1)^0 = (.9)^5 \approx .59$

(c) (5 points) Find the expected number of successes and the standard deviation.

$\mu =$ 4.5, $\sigma =$ .67

$(= 5 \times .9)$ $\quad (= \sqrt{5 \times .9 \times .1})$

(d) (5 points) Would 3 successes be an unusual outcome? Explain!

$\mu - 2\sigma = 4.5 - 2 \times .67 = 3.16 > 3$

so 3 is unusual.

3. (20 points) 1000 raffle tickets are sold at $5 per ticket. The prizes are:

First Prize: $2000
Second Prize: $1500
Third Prize: $1000

You buy 1 ticket. What is the expected value of your gain or loss?

		x	$P(x)$
lose	→	-5	$997/1000$
1^{st}	→	1995	$1/1000$
2^{nd}	→	1495	$1/1000$
3^{rd}	→	995	$1/1000$

$$E(x) = -5 \times 997/1000 + 1995 \times 1/1000 + 1495 \times 1/1000 + 995 \times 1/1000$$

$$= -500/1000 = \underline{-0.5}$$

Expected Loss of $0.50

4. The pregnancy lengths (in days) of humans are normally distributed with:

$$\mu = 268$$

$$\sigma = 15$$

Using the table provided, answer the following question:

What percent of pregnancies end at least two weeks early?

(a) (10 points) Find the z-score for 268 days minus 2 weeks using:

$$z = \frac{x - \mu}{\sigma}$$

(Be careful! You need to convert two weeks into days.)

$x = 268 - 14 = 254$

$z = \frac{254 - 268}{15} = -0.93$

(b) (10 points) Find the probability corresponding to this z-score using the table, and convert to percent by multiplying by 100%.

$P(z < -0.93) \sim 0.1762$

$\Rightarrow$ 17.62 %

5. What length of pregnancy corresponds to the 95th percentile?

(a) (10 points) Convert 95% into a probability and look up the z-score.

$0.95 \rightsquigarrow z = 1.645$

probability

(b) (10 points) Convert the z-score into a number of days using:

$$\mu = 268$$

$$\sigma = 15$$

and

$$x = z \cdot \sigma + \mu$$

$x = (1.645) \times 15 + 268$

$= \underline{293}$ (= up to 25 days late)