Name:____________________________________

Math 1040

Midterm Examination

March 22, 2016

Relax and good luck!

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. Consider the following speeds (in mph) of 20 cars on the freeway:

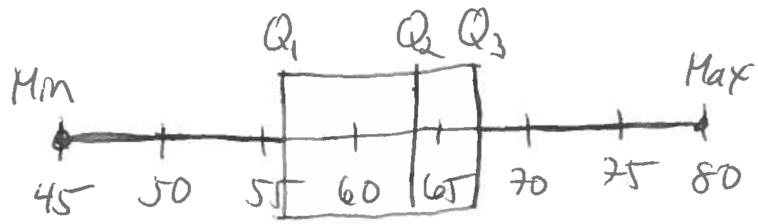
45 50 52 54 55 | 57 60 62 63 63 |

63 65 65 65 65 | 68 69 71 75 80

(a) (10 points) Find the three quartiles for the speeds:

Q1 = 56 **Q2** = 63 **Q3** = 66.5

(b) (5 points) Draw a box-and-whisker plot for the data:



(c) (5 points) What percentile corresponds to the speed limit of 70 mph?

# of speeds below 70 = 17

# of speed = 20

70 mph Percentile = 17/20 = 85%

2. The table below gives means and standard deviations for the heights of populations of men and women:

	Men's Heights	Women's Heights
Mean	69.9 in	64.3 in
Standard Deviation	3.0 in	2.6 in

(a) (5 points) Find the z-score for a 5' (= 60 inches) tall man.

$$z = \frac{x-\mu}{\sigma} = \frac{60-69.9}{3} = -3.3$$

(b) (5 points) Find the z-score for a 5' (= 60 inches) tall woman.

$$z = \frac{x-\mu}{\sigma} = \frac{60-64.3}{2.6} \approx -1.65$$

(c) (5 points) How tall (or short) must a man be to be considered unusual? (Recall that unusual is a z-score of more than 2 or less than -2).

$$z > 2 \Rightarrow \frac{x-69.9}{3} > 2 \Rightarrow x > 69.9 + 6 = 75.9'' \sim 6'4''$$

$$z < -2 \Rightarrow \frac{x-69.9}{3} < -2 \Rightarrow x < 69.9 - 6 = 63.9'' \sim 5'4''$$

(d) (5 points) How tall (or short) must a woman be to be considered unusual?

$$z > 2 \Rightarrow \frac{x-64.3}{2.6} > 2 \Rightarrow x > 64.3 + 5.2 = 69.5 \sim 5'9\tfrac{1}{2}''$$

$$z < -2 \Rightarrow \frac{x-64.3}{2.6} < -2 \Rightarrow x < 64.3 - 5.2 = 59.1 \sim 4'11''$$

3. Assume that the probability that a child is a girl is 1/2.
For a family with **four** children, what is the probability that:

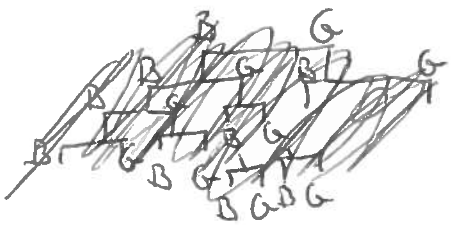
(a) (5 points) All the children are girls?

$$P(4 \text{ girls}) = P(\text{girl}) \times P(\text{girl}) \times P(\text{girl}) \times P(\text{girl})$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{16}}$$

(b) (5 points) One or more of the children is a girl?

$$P(\geq 1 \text{ girl}) = 1 - P(4 \text{ boys}) = 1 - \frac{1}{16} = \boxed{\frac{15}{16}}$$

(c) (5 points) Exactly one of the children is a girl?



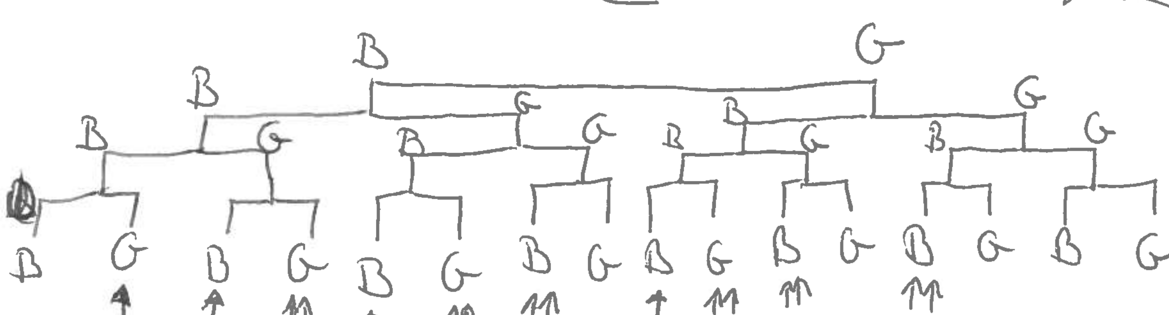
$$\boxed{4/16}$$

$$\frac{4 \uparrow}{16 \text{ total}}$$

(d) (5 points) Exactly two of the children are girls?

$$\boxed{\frac{6}{16}} \quad \frac{\uparrow\uparrow}{\text{total}}$$

Tree of possibilities



$\uparrow$ = one girl

$\uparrow\uparrow$ = two girls

4. The following table is the result of a survey of a total of 100 men and women asking them whether they were smokers or non-smokers:

	Non-Smoker	Smoker
Male	45	15
Female	30	10

(a) (3 points each) Find all the following empirical probabilities:

P(Smoker) = $25/100 = .25$

P(Male) = $60/100 = .60$

P(Male and Smoker) = $15/100 = .15$

P(Male or Smoker) = $70/100 = .70$

P(Smoker|Male) = $15/60 = .25$

(b) (5 points) According to the table, is being a smoker independent of being male? Explain your answer.

Since P(Smoker) = P(Smoker | Male), it follows that being a smoker is independent of being male.

5. A scholarship committee has **3** identical awards to give to top students. They are considering 15 applicants, 5 of whom are majoring in mathematics.

(a) (5 points) In how many different ways can they make the awards?

$$ {}_{15}C_3 = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} = 455 $$

(b) (5 points) In how many different ways can they make the awards, so that none of them go to math majors?

$$ {}_{10}C_3 = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120 $$

(c) (5 points) In how many different ways can they make them so that exactly one of them go to a math major?

$$ {}_{10}C_2 \times {}_5C_1 = \frac{10 \cdot 9}{2 \cdot 1} \times 5 = 225 $$

(d) (5 points) If the awards are made at random, what is the probability that one or more of them go to a math major?

$1 - P(\text{none to a math major})$

$= 1 - 120/455 \approx \underline{0.74}$