Name: KEY

Math 1040
Sample Final Examination

Relax and good luck!

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
5	25	
6	25	
7	25	
8	25	
Total	200	

1. (25 points) The systolic blood pressures of 20 elderly patients in a doctor's office were measured (in units of millimeters of mercury) with the results:

120 120 125 125 130 130 130 135 135 140

145 160 160 170 180 180 195 200 200 200

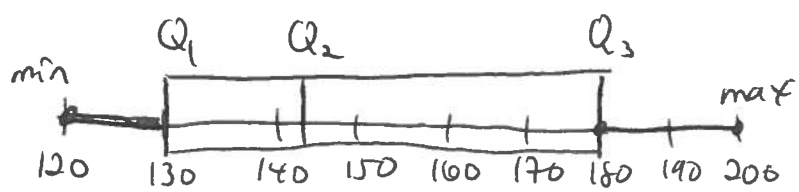
(a) (5 points) Calculate the mean, median and mode of the data:

Mean = 154 **Median** = 142.5 **Mode** = 130 and 200

(b) (5 points) Find the quartiles for the data:

Q1 = 130 **Q2** = 142.5 **Q3** = 180

(c) (5 points) Sketch the box and whiskers graph for the data:



(d) (5 points) What percentile represents a blood pressure of 150?

$11/20 = .55$

Percentile = 55

(e) (5 points) Name a blood pressure at the 20th percentile.

$4/20 = .20$

Pressure = 130

(or 129 or 128 or 127 or 126)

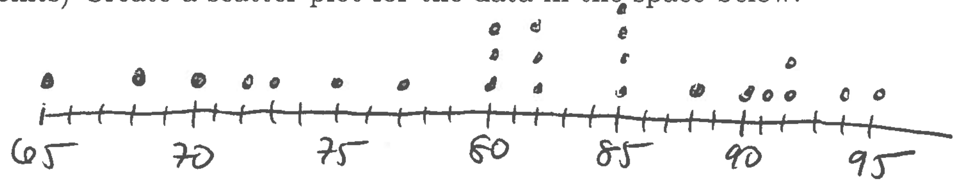
NOT 125 !

2. (25 points) The scores on a physics midterm exam (out of 100) are:

65 68 70 72 73 75 77 80 80 80 82 82

82 85 85 85 85 88 90 91 92 92 94 95

(a) (5 points) Create a scatter plot for the data in the space below:



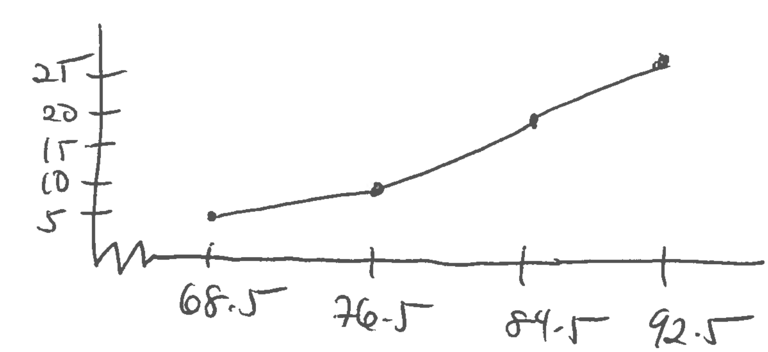
(b) (5 points) Create a stem-and-leaf plot for the data in the space below:

6	5 8
7	0 2 3 5 7
8	0 0 0 2 2 2 5 5 5 5 8
9	0 1 2 2 4 5

(c) (10 points) Fill out the table below, putting the data into **four** classes:

Classes	Midpoints	Frequencies	Cumulative Frequencies
65 – 72	68.5	4	4
73 – 80	76.5	6	10
81 – 88	84.5	8	18
89 – 96	92.5	6	24

(d) (5 points) Sketch an ogive for the four classes of data.



3. (25 points) You roll a pair of ordinary (6 sided) dice:

(a) (5 points) What is the probability that you roll "snake eyes"?

$1/36 \sim 0.03$

(b) (5 points) What is the probability that you roll a double of any kind?

$6/36 \sim 0.17$

(c) (5 points) What is the probability that you roll exactly a 7?

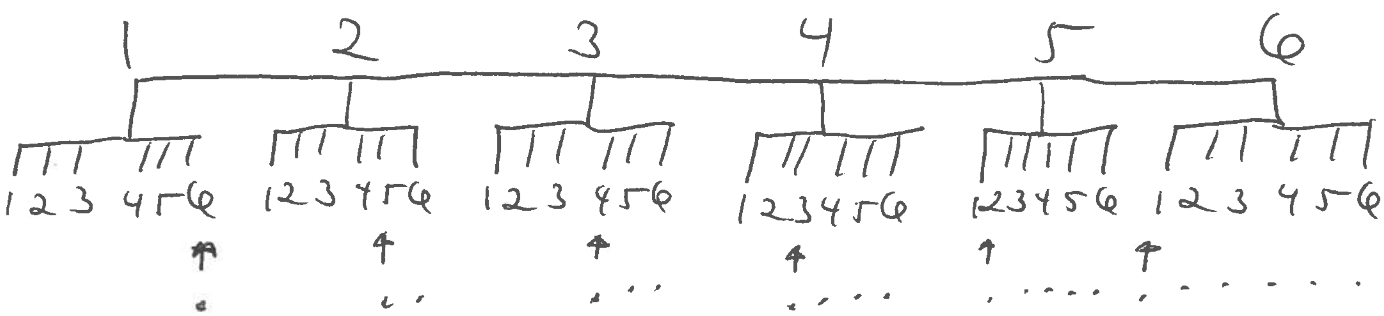
# of arrows = 6

$6/36 \sim 0.17$

(d) (10 points) What is the probability that you roll *at least* a 7?

to the right of the arrows (including the arrows)

$21/36 \sim 0.58$



4. (25 points) The table below breaks down the 432 members of the US House of Representatives of the 113th Congress by gender and political party:

	Republican (R)	Democrat (D)
Male	213	142
Female	19	58

Put your answers to all the questions below in decimal form:

(a) (5 points) Compute the following empirical probabilities.

$19+58=77$ $\quad$ $142+58=200$

P(Female) = $77/432 \sim 0.18$ **P(Democrat)** = $200/432 \sim 0.46$

(b) (5 points) Compute the following empirical probabilities:

$19+58+142=219$

P(Female and R) = $19/432 \sim 0.04$ **P(Female or D)** = $219/432 \sim 0.51$

(c) (10 points) Compute the following conditional empirical probabilities:

$213+19=232$

P(Female | R) = $19/232 \sim 0.08$ **P(Female | D)** = $58/200 \sim 0.29$

(d) (5 points) Is gender independent of party? Explain your answer!

$P(\text{Female}) = 0.18$

$P(\text{Female} \mid \text{Republican}) = 0.08$

These are not the same, so gender is not independent of party.

5. (25 points) The number of games played in the World Series from 1903 to 2012 are listed below together with their probabilities (the few early series that took 8 games are counted as 7).

x	$P(x)$	$x-\mu$	$(x-\mu)^2P(x)$
4	0.18	-1.77	0.56
5	0.24	$-.77$	0.14
6	0.21	$.23$	0.01
7	0.37	1.23	0.56

(a) (15 points) Fill out the rest of the table, and calculate:

$$\mu = \sum x \, P(x) = 4 \times 0.18 + 5 \times 0.24 + 6 \times 0.21 + 7 \times 0.37$$

$$= 5.77$$

$$\sigma = \sum (x-\mu)^2 P(x) = .56 + .14 + .01 + .56 = 1.27$$

Expected Number of Games $\mu = \underline{5.77}$ $\quad\sigma = \underline{1.27}$

(b) (5 points) What is the z-score of a 4 game world series?

$$z = \frac{x-\mu}{\sigma} = \frac{4-5.77}{1.27} = -1.39$$

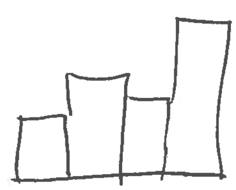
(c) (5 points) Look up the probability associated to the z-score in (b) and compare it with the probability of a 4-game world series from the table.

Why are the two probabilities different?

Table probability $= 0.08$

$P(4) = 0.18$

The actual probability distribution is:



which is not close to a normal distribution. The table gives probabilities for normal distributions!

6. (25 points) "The Google" says that:

222 out of 365 days of the year are sunny in Salt Lake City

In other words, the probability that a given day is sunny is:

$$222/365 = 0.61$$

(a) (5 points) You perform the binomial experiment of looking at the sky every day for one week, and jot in your notebook whether the day was sunny. Assuming that "success" is a sunny day, answer the following:

$n =$ 7 $\quad p =$ .61 $\quad q =$ .39

(b) (5 points) What are the expected number of sunny days and the standard deviation?

$\mu =$ 4.27 $\quad \sigma =$ 1.29

$\mu = np = 7 \times .61$

$\sigma = \sqrt{npq} = \sqrt{7 \times .61 \times .39}$

(c) (5 points) Using the formula:

$$_nC_x = \frac{n!}{x!(n-x)!}$$

calculate:

$_7C_5 =$ 21

$$\frac{7!}{5! \times 2!} = \frac{7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{2 \cdot 1 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{42}{2}$$

(d) (5 points) What is the probability that exactly 5 days are sunny?

$\mathbf{P(5)} =$ 0.27 $\quad P(5) = {}_7C_5 \times p^5 \times q^2 = 21 \times (.61)^5 \times (.39)^2$

(e) (5 points) Calculate the z-score for **zero** sunny days, and decide whether or not this is an unusual occurrance in Salt Lake City.

$\mathbf{z} =$ −3.31 $\quad$ Unusual? Y/N Y

(A z-score less than −2 is unusual!)

$$z = \frac{x-\mu}{\sigma} = \frac{0-4.27}{1.29} = -3.31$$

7. The mean height and standard deviation for adult women (in inches) are:

$$\mu = 64.3$$

$$\sigma = 2.6$$

Using the table provided, answer the following questions:

(a) (5 points) Find the z-score of a 6 foot woman (convert to inches!)

6 ft = $6 \times 12 = 72''$

z = 2.96

$$z = \frac{x - \mu}{\sigma} = \frac{72 - 64.3}{2.6}$$

(b) (10 points) Find the percentile for a 6 foot tall woman.

.9985

Percentile = 99.85

(c) (10 points) Find the height of a woman who is at the 40th percentile.

Height = $63.65'' \sim 5''3.65''$

40^{th} percentile $\Rightarrow P(z) = 0.40$

$\Rightarrow z = -0.25$

$x = z \cdot \sigma + \mu = (-.25)(2.6) + 64.3 =$

8. (25 points) Ordered pairs of data were collected below, comparing the mean daily intake of calories of various countries (in thousands, x) with the infant mortality rate (per one hundred births, y).

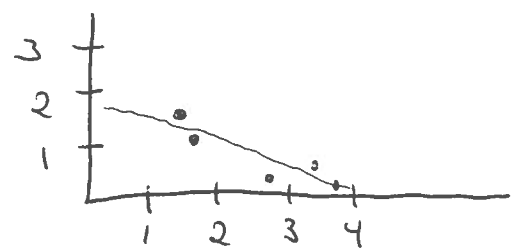
(a) (5 points) Complete the following table:

5

x	y	x^2	y^2	xy
1.5	1.5	2.25	2.25	2.25
2.7	0.2	7.29	0.04	0.54
1.6	1.1	2.56	1.21	1.76
3.7	0.1	13.69	0.01	0.37
3.4	0.4	11.56	0.16	1.36
12.9	3.3	37.35	3.67	6.28

$\leftarrow \Sigma$

(b) (5 points) Plot the data:



$$m = \frac{(5)(6.28)-(12.9)(3.3)}{(5)(37.35)-(12.9)(12.9)}$$

~ -0.55

$b = 0.66 - (-.55)(2.56)$

$= 2.08$

(c) (5 points) Circle the most likely correlation coefficient for the data:

Almost on a line of negative slope!

$r = -0.9$ $\qquad r = -0.1 \qquad r = 0.1 \qquad r = 0.9$

(d) (5 points) Find the equation for the line of regression for the data.

$$y = (-0.55)x + 2.08$$

(e) (5 points) How many infants out of 100 would you expect not to survive in a country whose daily intake of calories is 2000?

$2000 \rightsquigarrow x = 2$

$$y = (-0.55)(2) + 2.08$$

$$= \underline{0.98}$$

$\boxed{\underline{0.98} \text{ mortality}}$

The Formula Page

Mean (of n individual data items labeled with x):

$$\mu = \overline{x} = \frac{\sum x}{n}$$

Standard Deviation (of n individual data items labeled with x)

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{n}}$$

Expected Value (of data items labeled with x with probabilities $P(x)$)

$$\mu = E(x) = \sum xP(x)$$

Standard Deviation (of data items labeled with x with probabilities $P(x)$)

$$\sigma = \sqrt{\sum(x-\mu)^2 P(x)}$$

A Binomial Experiment with n trials and probability of success p has:

Probability of failure: $q = 1 - p$.

Probability of exactly x successes (out of n trials): ${}_nC_x p^x q^{n-x}$

Expected number of successes: $\mu = np$

Standard deviation: $\sigma = \sqrt{npq}$

Conversion from x data to z-scores (and back):

$$z = \frac{x-\mu}{\sigma} \text{ and } x = z\sigma + \mu$$

The Line of Regression of n ordered pairs labeled with (x, y) is:

$$y = mx + b$$

with

$$m = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)(\sum x)}$$

and

$$b = \frac{\sum y}{n} - m \cdot \frac{\sum x}{n}$$