Mathematical Modeling of Liver Injury and Dysfunction After Acetaminophen Overdose: Early Discrimination Between Survival and Death

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Acetaminophen (APAP) is the leading cause of acute liver injury in the developed world. Timely administration of N-acetylcysteine (N-Ac) prevents the progression of serious liver injury and disease, whereas failure to administer N-Ac within a critical time frame allows disease progression and in the most severe cases may result in liver failure or death. In this situation, liver transplantation may be the only life-saving measure. Thus, the outcome of an APAP overdose depends on the size of the overdose and the time to first administration of N-Ac. We developed a system of differential equations to describe acute liver injury due to APAP overdose. The Model for Acetaminophen-induced Liver Damage (MALD) uses a patient’s aspartate aminotransferase (AST), alanine aminotransferase (ALT), and international normalized ratio (INR) measurements on admission to estimate overdose amount, time elapsed since overdose, and outcome. The mathematical model was then tested on 53 patients from the University of Utah. With the addition of serum creatinine, eventual death was predicted with 100% sensitivity, 91% specificity, 67% positive predictive value (PPV), and 100% negative predictive value (NPV) in this retrospective study. Using only initial AST, ALT, and INR measurements, the model accurately predicted subsequent laboratory values for the majority of individual patients. This is the first dynamical rather than statistical approach to determine poor prognosis in patients with life-threatening liver disease due to APAP overdose. Conclusion: MALD provides a method to estimate overdose amount, time elapsed since overdose, and outcome from patient laboratory values commonly available on admission in cases of acute liver failure due to APAP overdose and should be validated in multicenter prospective evaluation. (HEPATOLOGY 2012;00:000–000)
usually unaware of the timing or the dose of drug taken, and concomitant use of other medications or drugs often obscures the clinical picture.

We therefore sought a method for rapidly determining the time of overdose, extent of injury, and likelihood of spontaneous survival using laboratory data available at the time of admission. Our method is based on a mathematical model that describes typical hepatic injury progression, dependent only on overdose amount. Fitting patient laboratory values to our mathematical model allows for the estimation of overdose amount and timing, as well as a prediction of outcome. We tested the mathematical model on 53 patients from the University of Utah.

Materials and Methods

Model Background. Our mathematical model, the Model of Acetaminophen-induced Liver Damage (MALD), is based on a reproducible pattern of APAP-induced liver injury. The enzymes aspartate aminotransferase (AST) and alanine aminotransferase (ALT) are released by injured hepatocytes. These enzymes peak at about 36 hours from initial injury and have distinct injury and clearance curves. AST concentration in blood is initially approximately double that of ALT, with a clearance rate of about 50% every 24 hours. ALT peaks at about the same time as AST, but with a slower elimination rate of about 33% every 24 hours. These measures of damage are complemented by a measure of liver function, prothrombin time/international normalized ratio (INR). Decreased production of essential clotting factors manifests as reduced clotting and increased INR, again with characteristic rates of increase and decay. The values of AST, ALT, and INR at the time of admission thus encode the course of disease progression over time and can be used, with a suitable mathematical model, to estimate initial dose and time of overdose.

Model Description. We developed a system of nonlinear ordinary differential equations to describe the temporal dynamics of APAP-induced acute liver failure (ALF) based on known mechanisms of APAP metabolism (Supporting Information). The equations describe NAPQI production from APAP metabolism, glutathione conjugation, hepatocyte death by NAPQI, release and clearance of AST and ALT in the blood, hepatocyte regeneration, and clotting factor production (Fig. 1). The variables and parameters can be divided into those describing hepatocyte, APAP, glutathione, INR, and AST/ALT dynamics.

Functional hepatocytes \( (H) \) become damaged hepatocytes \( (Z) \) and regenerate with the following parameters:

- The number of hepatocytes in a healthy liver is \( H_{\text{max}} = 1.6 \times 10^{11} \) cells.\(^\text{12,14}\)
- Damaged hepatocytes lyse with rate \( \delta_z = 5/\text{day} \).
- Functional hepatocytes regenerate with rate \( r = 1/\text{day} \).\(^\text{15}\)
- Functional hepatocytes become damaged with rate \( \eta = 5.12 \times 10^{15} \text{cell/mol/day} \).
- The fraction of liver required for survival is \( \mu = 0.3 \).\(^\text{16}\)

Serum APAP \( (A) \) is a surrogate for liver APAP, which is converted to NAPQI \( (N) \) with the following parameters:
• APAP is cleared by hepatocytes with rate \( \alpha = 6.3/\text{day}. \)\(^{17} \)
• APAP is cleared unconjugated with rate \( \delta_a = 0.33/\text{day}. \)\(^{2,3} \)
• The fraction of APAP that is oxidized to NAPQI is \( p = 0.05. \)\(^{2,3} \)
• The conversion factor from grams of APAP to mol of NAPQI is \( q = 0.0067 \text{ mol/g}. \)

GSH (G) is associated with the following parameters:

• GSH binds to NAPQI with rate \( \gamma = 1.6*10^{18} \text{ cell/mol/day}. \)\(^{18} \)
• GSH decays with rate \( \delta_g = 2/\text{day}. \)\(^{19,20,21} \)
• GSH is produced with rate \( \kappa = 1.375*10^{-14} \text{ mol/cell/day}. \)

INR (I) is related to the clotting factor concentration as a fraction of normal (F) and is associated with the following parameters:

• Clotting factor VII is cleared with rate \( \beta_f = 5/\text{day}. \)\(^{22} \)
• The minimum clotting factor concentration is \( F_{\text{min}} = 0.75. \)

Serum AST concentration (S) and serum ALT concentration (L) increase and decay with the following parameters:

• AST is cleared with rate \( \delta_s = 0.92/\text{day}. \)\(^{12} \)
• ALT is cleared with rate \( \delta_l = 0.35/\text{day}. \)\(^{12} \)
• The total amount of AST in a healthy liver is \( \beta_s = 200,000 \text{ IU}. \)
• The total amount of ALT in a healthy liver is \( \beta_l = 84,800 \text{ IU}. \)
• The amount of blood in a human body is \( v = 5 \text{ L}. \)
• The minimum AST level is \( S_{\text{min}} = 12 \text{ IU/L}. \)
• The minimum ALT level is \( L_{\text{min}} = 9 \text{ IU/L}. \)

Six parameters were adjusted to match properties of the data, independent of patient survival information. The amounts of AST and ALT in the liver, \( \beta_s \) and \( \beta_l \), respectively, were scaled to the maximum observed AST and ALT values, and the minimum AST and ALT levels, \( S_{\text{min}} \) and \( L_{\text{min}} \), respectively, were scaled to the minimum observed AST and ALT values. The minimum clotting factor concentration \( F_{\text{min}} \) was scaled to the maximum observed INR value. The damaged hepatocyte lysis rate \( \delta_c \) was adjusted to the timing of peak AST and ALT values.

Two parameters were scaled to the dose of APAP required for hepatotoxicity and death. The glutathione production rate, \( \kappa \), was scaled to the dosage at which glutathione reserves are depleted. The minimum dosage predicted to lead to hepatotoxicity varies, but typically ranges from 7.5 to 10 g for an adult.\(^{6,23} \) We chose a slightly lower value of 6.0 g for the dosage at which glutathione reserves are depleted. The rate at which hepatocytes become damaged by NAPQI, \( \eta \), is a scaling factor that was chosen so that a 20 g overdose is equivalent to 70% hepatic necrosis and predicted death.

Patients. Between January 1, 2006, and December 31, 2009, all hospital discharges from the University of Utah were queried for the diagnosis of severe, acute APAP toxicity. Charts were excluded if they included acute hepatitis A or B, autoimmune hepatitis, Wilson Disease, or multisystem failure. Laboratory data and admission and discharge notes were further reviewed to identify cases in which acute liver disease was due to APAP overdose only. Charts that had overdose with additional medications were not included in this analysis. Demographics, N-Ac administration, and medical outcome information were collected. Laboratory results of AST, ALT, INR, bilirubin, and creatinine were also collected. Charts without at least one measure of AST, ALT, and INR were excluded from the study. In total, 53 patients were included. The patient population was diverse, with varying alcohol use, body mass index, and ingestion type, including suicide attempts, single accidental overdoses, and multiple day chronic overdoses.

Ethics Statement. Patient consent was not obtained because data were retrospective, were based on standard care, and were analyzed anonymously. The protocol was approved by the Institutional Review Board (IRB) of the University of Utah in accordance with the Declaration of Helsinki.

Serum Creatinine. Serum creatinine was added as an additional criterion separate from the model because it is a marker of kidney damage and our dynamic model does not describe kidney damage. Because kidney function is ultimately important in survival in APAP overdose, patients with serum creatinine greater than 3.4 mg/dL were predicted to die.\(^{24} \)

Fitting the Model to Individual Patients. Upon admission, before administration of N-Ac, a patient’s AST, ALT, and INR values in the mathematical model are a function of two parameters, APAP overdose amount, \( A_0 \), and time since overdose, \( \tau \). These two parameters were estimated using weighted least-squares and values of AST, ALT, and INR on admission. The weights were determined by posttreatment model fits (see Supporting Information for more details). To test
the sensitivity of predicted outcomes to changes in parameters, we increased and decreased each parameter by 50% of its original value and fit individuals to the model, keeping track of the predicted outcome for each patient.

Results

We tested the model on 53 patients from the University of Utah. The time since overdose and overdose amount were estimated for each patient using initial measurements of AST, ALT, and INR on admission (Fig. 2). Based on the extent of estimated liver injury, the model predicts death for patients who took over 20 g of APAP without N-Ac administration within the first 24 hours.

Excluding patients who were transplanted, death versus recovery was predicted with 75% sensitivity and 95% specificity (Table 1). With the addition of initial serum creatinine exceeding 3.4 mg/dL on admission (Fig. 2), sensitivity increased to 100%. For this dataset the subset of the King’s College Criteria (KCC) to which we had access (INR > 6.5 and creatinine > 3.4 mg/dL) had 13% sensitivity and 100% specificity. Only one patient had both INR > 6.5 and creatinine > 3.4 on admission. Thinking of the KCC as either INR > 6.5 or creatinine > 3.4 mg/dL increased sensitivity to 88%. We did not have access to patient encephalopathy or arterial pH.

Using only data available on admission, the model results fit the posttreatment time-series of the markers of liver damage for the majority of individual patients (Supporting Information Table 2). The results from four representative patients are shown in Fig. 3. Patients 5 and 8 were predicted to have had overdoses that were very close to the lethal threshold, whereas patient 49 was predicted to have exceeded the lethal threshold. Patient 16 was predicted to have had a smaller overdose. The confidence region for some patients who recovered (e.g., patient 16) includes regions with high overdose amount and very early N-Ac administration, as well as regions with low overdose amount and late N-Ac administration. In both cases AST, ALT, and INR are low.

Model predictions of outcome were robust to 50% increase or decrease in parameter values (Supporting Information Table 3). The most sensitive model parameters were the fraction of liver required for survival, \(l\), and the amount of AST in the liver, \(b_s\). Increasing \(l\) to 0.45 caused more patients who eventually recovered to be predicted to die, and resulted in 100% sensitivity and 77% specificity, whereas decreasing \(l\) to 0.15 resulted in 88% sensitivity and 93% specificity. Increasing \(b_s\) by 50% resulted in 100% sensitivity and 79% specificity, whereas decreasing \(b_s\) by 50% resulted in 88% sensitivity and 88% specificity.

Some parameters such as \(p\), the fraction of APAP oxidized to NAPQI, have a large effect on predicted dose of APAP, but no effect on predicted outcome. If \(p\) is 0.025, an overdose amount of 40 g is required for 70% hepatic necrosis and predicted death, whereas if \(p\) is 0.075, an overdose amount of 13.3 g is required for...

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**Table 1. Sensitivity, Specificity, PPV, and NPV for a Subset of King’s College Criteria (INR > 6.5 and Creatinine > 3.4 mg/dL), Either INR > 6.5 or Creatinine > 3.4 mg/dL, and the Current Study Both With and Without Creatinine as an Independent Marker**

<table>
<thead>
<tr>
<th>Model</th>
<th>Sensitivity</th>
<th>Specificity</th>
<th>PPV</th>
<th>NPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>INR &gt; 6.5 and creatinine &gt; 3.4 mg/dL</td>
<td>0.13 (1/8, 0.0-0.53)</td>
<td>1 (43/43, 0.92-1)</td>
<td>1 (1/1, 0-1)</td>
<td>0.86 (43/50, 0.73-0.94)</td>
</tr>
<tr>
<td>INR &gt; 6.5 or creatinine &gt; 3.4 mg/dL</td>
<td>0.88 (7/8, 0.47-1)</td>
<td>0.95 (41/43, 0.84-0.99)</td>
<td>0.78 (7/9, 0.4-0.97)</td>
<td>0.98 (41/42, 0.87-1)</td>
</tr>
<tr>
<td>MALD (no creatinine)</td>
<td>0.75 (6/8, 0.35-0.97)</td>
<td>0.95 (41/43, 0.84-0.99)</td>
<td>0.75 (6/8, 0.35-0.97)</td>
<td>0.95 (41/43, 0.84-0.99)</td>
</tr>
<tr>
<td>MALD (with creatinine)</td>
<td>1 (8/8, 0.63-1)</td>
<td>0.91 (39/43, 0.78-0.97)</td>
<td>0.67 (8/12, 0.35-0.90)</td>
<td>1 (39/39, 0.91-1)</td>
</tr>
</tbody>
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Absolute numbers and 95% Clopper-Pearson confidence interval are given in parentheses.
70% hepatic necrosis and predicted death. Estimates of overdose amount scale with lethal dose so that estimates of outcome remain the same despite large changes in estimated overdose amount.

**Discussion**

APAP, alone or in combination, accounts for about 50% of cases of ALF in the USA. Survival largely depends on two parameters: the size of the initial dose and time elapsed prior to the administration of N-Ac. Very early administration (up to 12 hours after overdose) of N-Ac results in almost 100% survival. Some models of APAP toxicity rely on the time between ingestion and hospital admission to determine the need for treatment or as a measure of exposure. These are risky approaches because the timing of the overdose provided by the patient is frequently unobtainable or unreliable. Moreover, patients who arrive at the hospital 24 hours or more postingestion may have plasma APAP levels below the detection limit.

The KCC provides a well-validated method for predicting death without transplantation in APAP-induced ALF, although they have been criticized for low sensitivity and low negative predictive value (NPV). KCC used an initial dataset of 310 patients
to identify statistically significant prognostic indicators
to distinguish survivors and nonsurvivors and used a
validation set of 121 patients to identify cutoff values
associated with survival rates less than 20% for the
statistically significant prognostic indicators, with no
physiologically defined model of mortality. Many
modifications of the KCC have been suggested,\textsuperscript{30–35}
perhaps most importantly the addition of arterial lact-
tate.\textsuperscript{36} Arterial lactate has consistently been shown to
be associated with survival, although its prognostic
value has been questioned.\textsuperscript{37}

In contrast to other modifications of the KCC,
MALD is novel because we build upon the KCC by
utilizing an understanding of the dynamics of hepato-
cyte damage following APAP overdose in the form of a
dynamic mathematical model. Hepatic necrosis is
directly related to the extent of covalent binding of
NAPQI to intracellular components,\textsuperscript{2,4,6,7} which causes
hepatocyte lysis and release of AST and ALT into the
blood. This produces a characteristic time course of
injury with an early rise and predictable decay of AST,
ALT, and INR. We have developed a system of differ-
ential equations based on the principles of APAP-
induced liver damage. All parameters in MALD were
estimated from the literature, except six that were
adjusted to match general properties of AST and ALT
dynamics, and two that were scaled to the dosages
thought to cause hepatotoxicity and death. Survival
information from University of Utah patients was not
used in model development or parameterization. The
equations describe how AST, ALT, and INR levels
change over time as a function of overdose amount.
Because these curves over time are only a function of
initial overdose amount, AST, ALT, and INR levels in
the model only depend on initial overdose amount
and time since overdose. Our method works by fitting
measured AST, ALT, and INR values to the curves
described by our differential equations to estimate
overdose timing and amount (Fig. 4). An outcome of
death is predicted when the estimate of overdose
amount is sufficiently high and the estimate of timing
predicts N-Ac to be ineffectual, or when serum creati-
nine measurements are sufficiently high. If the out-
come is predicted to be poor, liver transplantation may
be the only life-saving treatment.

Previous studies have not found absolute aminotrans-
ferase levels to be significant predictors of outcome in cases of APAP-induced ALF.\textsuperscript{24} This is not
surprising because aminotransferase levels will be low,
even with a high dose, both early and late in the
course of the injury based on known mechanisms of
liver damage following APAP overdose. Similarly, high
aminotransferase levels may be measured near peak
liver damage, even in cases of nonlethal overdose. In
conjunction with INR and a suitable mathematical
model describing these mechanisms, however, amino-
transferase levels do contain sufficient information to
estimate the timing and amount of overdose.

Our model cannot distinguish patients with high
overdose amounts and early administration of N-Ac
from patients with low overdose amounts and delayed
treatment because in both cases AST, ALT, and INR
levels are low. However, this ambiguity affects only
patients who are predicted to recover.

Some patients with unique characteristics, such as
those with significant muscle damage, may not fit the
model. Muscle damage increases the level of AST,
which may lead to poor estimation of liver damage.
Because ALT and INR values are not affected by mus-
cle damage, this effect may be minimal. Further stud-
ies are warranted to determine whether more refine-
ments are needed for special patient groups.

Our treatment of all patients as having the same
parameter values is unrealistic. Well-known covariates
do disease severity such as age,\textsuperscript{38} chronic alcohol
use,\textsuperscript{39,40} starvation or malnutrition,\textsuperscript{41} and interactions
with other drugs\textsuperscript{42,43,44} may affect the parameter val-
ues of an individual. In some cases these differences
will not affect the accuracy of predictions of outcome.
Model predictions derive from the amount of uncon-
jugated NAPQI that results from a given dose, but
that amount may depend on patient characteristics.
For example, alcoholics may make excessive NAPQI because of elevated p-450 levels, or individuals may have decreased levels of GSH because of starvation, competition from other drugs, or genetic variation. These differences might make the model estimates of initial dose seem overly high, but the outcome could still be accurately predicted because these patients have more unconjugated NAPQI than is typical for the overdose amount.

James et al.\(^4\)\(^5\) show that APAP protein adduct levels may be used as specific biomarkers of APAP toxicity. If measurements were routinely available, adducts could easily be added to our model, and might provide additional predictive value. However, the correlation of protein adducts with AST and their similar kinetics lead us to predict this effect would be small, although their more direct relationship to liver damage might reduce noise and make them a superior predictor.

Gregory et al.\(^4\)\(^6\) found that individuals with overdose amounts greater than 10 g did not have significantly different mortality than those reporting smaller overdoses in patients with eventual hepatic encephalopathy. The authors suggest that this may be due to inaccurate reporting of dosing information by patients with eventual hepatic encephalopathy, or from a plateau effect in APAP overdose amount, such that above a threshold the effect of APAP overdose ceases to be additive. A plateau is built into our model, but at 20 g rather than 10 g. In our model, without treatment, any overdose above 20 g will result in severe hepatic injury, maximal AST, ALT, and INR levels, and poor outcome. Our patient set is quite different because Gregory et al. required eventual hepatic encephalopathy for inclusion, a parameter unknown on admission and associated with poor prognosis.\(^4\)\(^7\)

Methods to determine whether to use dangerous and costly interventions, such as transplantation, will ideally be based on clinical data that are readily available at the time of admission. Using only initial measurements of AST, ALT, and INR, we were able to predict the hepatic injury progression and extent of liver damage following APAP overdose. Unlike statistical models to predict outcome, which must build on survivorship data, our mechanistic approach is based on the independently testable assumption that 70% hepatic necrosis leads to death. Our dynamic model yields a prediction of outcome by estimating the time since overdose and overdose amount from commonly obtained laboratory data on admission. With the inclusion of creatinine, we were able, in this retrospective analysis, to predict survival versus death with 91% specificity, 100% sensitivity, 67% PPV, and 100% NPV. Our initial analysis suggests that MALD compares favorably to statistical methods, and should be validated in multicenter retrospective and prospective evaluation.

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