Minimizing the Free Energy of "Metallic" Spheres in ER Fluids Continued II
Final Report Fall 2006

This past semester, we investigated the role of the Yang-Lee Theorem in the Ising Model of Statistical Mechanics. We studied the derivation of the Hamiltonian of the system to the Partition Function and from there focused on how this gives us a function for the Free Energy of the system. From an integral representation of the free energy, we found the magnetization or the effective conductivity of the system. To gain a better intuition of the Yang-Lee Theorem a Matlab computer program was written to show that the roots of the Partition Function lie on the unit circle in the complex plane and that as we increase the number of spins in the Ising Model the zeros begin to "pinch" the positive real axis denoting a phase transition. We learned the basics to statistical mechanics in order to be able to apply them to macroscopic composite systems such as Electrorheological Fluids.

Electrorheological (ER) Fluids consist of glass spheres suspended in oil. When one passes an electric field through this medium which exceeds a critical strength the spheres align themselves into columns parallel to the direction of the electric field in a matter of milliseconds. These composites have many applications, some of them being to the car industry and the health industry in making artificial knees. The rapid change in configuration reminds us of phase transitions in statistical mechanics although the process is not temperature driven. Using the Yang-Lee Theorem of the Ising model for a ferromagnet as a link, we can apply ideas from statistical mechanics to composite materials to help understand the ER transition.