

Spring 2008 VIGRE Report

With relatively recent discoveries in mathematical approaches to pricing financial instruments, much research has been performed seeking to create more accurate models to be used for investment purposes. One of the areas studied is how to price debt, often in the form of bonds, which companies sell in attempts to raise capital with the promise of interest payments of some sort. This investment involves risk due to the chance that a company may go bankrupt and pay back less than the total owed debt, or possibly none at all. There are indicators which may be used to measure a company's riskiness, also called credit ratings, which are independently created by organizations such as Moody's or Standard and Poor's. They measure the quality of the debt issued by companies, particularly the likelihood of default for a company. This helps investors decide how much should be paid for a given bond. However, an impediment to analyzing the credit risk is the variable nature of firms' credit ratings, which fluctuate with time. The ratings can change as the company does or does not have the earnings they were expecting for that year, or as consumer confidence changes. Theories of mathematical finance may be used to price derivatives on financial securities, thereby giving investors information on the riskiness of an investment. Some of the means by which the prices of these derivatives is determined will be discussed, along with areas where some improvements may be made.

In the previous report of last semester's project, I discussed the models which are used to describe the stochastic nature of credit risk. These models were useful in determining the course that the research should take, though it was not enough to do exactly what was intended. Once again, Professor Jingyi Zhu was the supervisor to my VIGRE research, helping to continue our earlier work. Still with the goal in mind of showing business-cycle related changes, a different method was required. It would have to still be reminiscent of the previous time-dependent generator method,

which is better than a constant generator method. The constant generator method assumes identical transitions between each time period. However, this is definitely not the case. Each step of time must have a different matrix, which reflects the fact that the state of the different firms is constantly in flux. Inspiration for the new method came after doing more research and finding a paper by William Perraudin. [1] His work at the Bank of England described transition matrices which were used to describe changes which occurred during different portions of the business cycle, the trough, peak, and normal times. This was incredibly fortuitous, since access to information about the states of firms over time is limited and costly. Using this transition matrix, I transformed it into a generator matrix to create a new transition matrix which allowed for unobserved transitions. [2] This process was also described in the previous semester's report, and will not be included here. This gave three different matrices which could be used to simulate time-dependence. The matrices would no longer be constant, so this would help prevent the problems associated with a homogeneous Markov process trying to simulate a nonhomogeneous process.

This was followed by some very interesting results. Using the Matlab code which had been created last semester, it was not difficult to integrate this new information. Using the transition matrices, the initial spread was used to represent the price of things as they now were, and then the three states of the business cycle were used to alter it. First, I sought to match the data we had, which could be done by using the normal transition matrix in all but the third year, which was a peak year. This gave some very nice results which were surprisingly close to the data we already had. I did some more research into the business cycle, learning from the National Bureau of Economic Research about the timing of peaks and troughs.[3] It was very interesting to notice after some analytical work that the trough followed the peak within a year almost without fail. Some were shorter by a quarter, and some later by a quarter, but almost always a year was the length of a recession. I used this data to create a theoretical model of the credit spreads. I supposed that

the economy would be at a peak right now, and then it would be followed in the next year by a trough, then normal times for the next three years. This gave results which were not reflected at all in the data.

This leads to the following conclusion, fairly speculative though supportable from the observations. It seems that the prediction for credit spreads based on the current state of the business cycle is done without regard to which state it currently is in. Logically, people think they are in a 'normal time', and therefore base their decisions off of that. It may be possible to use the extra information if it is quite evident that the economy is in a recession, or peaking in a bubble. If this is so, then arbitrage opportunities may become evident by having this extra information. This allows one to conclude that if it were possible to use the information of the state currently being experienced, then using this model may lead an individual to discover an arbitrage opportunity.

Because of this, it seems worthwhile to research further into the topic to discover whether or not this business cycle model is useful. It may be possible to examine old information on credit spreads during a known trough or peak, and use that information to calibrate the model. Then it could be used in the present to purchase bonds when the state of the business cycle is 'known'. The trick here is whether or not the individual really can know where they are in the business cycle, or how long it will really take before another recession or peak. It may also take more or less time for the recession to end, or for the normal times to return again. It would be beneficial to study the relative credit ratings of the different firms in order to better define the position of the economy in the business cycle. In peak times, firms default less often, but even top firms stay at AAA from one transition to the next less often. In the worst times, firms obviously do much worse, and defaults are more frequent. In the normal times, most firms stay about in the middle, with a definite central tendency. A way to measure the position of the business cycle which the economy is in would be very beneficial to companies seeking this sort of insight into such a model.

The other item of interest would be to create a continuous model, not one that is discretized in 1-year increments. It is simple to create a somewhat continuous model with 3-month or 1-month increments, however the ability to match a model would be made more difficult. The reason for this is the fact that bond spreads are only measured in 1-year intervals, not monthly or quarterly.

This problem has really come together in the last couple of months. The information from the paper written by the man in the Bank of England was also very helpful. If possible, I would like to consider finishing this research and presenting it at Mathfest this summer. I don't think there is enough work to continue for another semester, however, it may be worthwhile to present it this summer. It has been very interesting to learn about, and hopefully will help me sometime in my future career as a financial engineer.

References

- [1] Nickell, P., Perraudin, W., Varotto, S., Stability of rating transitions, *Journal of Banking and Finance*, 2000.
- [2] Lando, D., 2004, *Credit Risk Modeling: Theory and Applications*, Princeton University Press, New Jersey.
- [3] www.nber.org