REU PROJECT PROPOSAL

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The concept of a basis for a category of objects is a fundamental principle in mathematics. In many cases, an orthonormal basis is a good choice for representing something, such as constructing a function with a Fourier Series. Often, however, the most useful way to compose a function or vector is not with an orthonormal basis but instead with a repetitious collection of vector or function parts. These over-complete spanning systems are called frames, and have some remarkable properties and applications.

Formally, a frame is a (redundant) set of vectors $\{\phi_n\}_{n\in I}$ in a Hilbert space H if there are numbers $A, B \geq 0$ such that for all $\phi \in H$.

$$A\|\phi\|^2 \leq \sum_{n\in I} |\langle \phi, \phi_n \rangle|^2 \leq B\|\phi\|^2.$$

When further conditions are applied to the frame bounds A and B, and when the vectors satisfy some other criteria, the frame exhibits some extremely useful properties. Engineers and scientists have found them to be a very handy tool, and examples of their application include signal processing, image compression, data transmission, and sampling. In some cases of signal decomposition, it is actually impossible to construct an orthonormal basis with time-frequency localization, but it is relatively easy to come up with a frame that fits the bill.

Working in concert with Dr. Nathan Smale, I intend to first establish a solid foundation in linear algebra, particularly finite and infinite Hilbert spaces and operators over these spaces. At this point, I will study some of the varieties of frames, including Parseval frames, Gabor frames, and affine (wavelet) frames. Some instances of applications of these will be considered. Finally, I will examine Grassmannian frames and attempt to develop some kind of algorithmic method to construct them. Frame theory is an exciting and challenging area of functional analysis and I enthusiastically await the opportunity to explore it, in addition to making a first attempt at mathematical research.