

Calculus II Practice Final

There are 12 questions; each question is worth 10 points. Do any ten of them. There is a final problem worth 10 points of extra credit.

1. Differentiate:

a) $f(x) = \ln(\sin(e^{2x}))$.

b) $g(x) = x \tan^{-1}(x^2)$.

c) $h(x) = e^{\ln x}$.

2. Find the integrals:

a) $\int u^2(u-1)^5 du$

b) $\int x(\ln x) dx$

c) $\int \frac{e^x}{1+e^x} dx$

3) $\int \frac{3x+1}{x(x^2+1)} dx$

4) $\int \frac{x^2+1}{(x-1)(x-2)(x-3)} dx$

5) $\int e^x \sin x dx$

6. The population of Dim Corners, Alabama has been decreasing at a rate of 4.6% per year for the past ten years. If the present population is 6,100, what was the population six years ago?

7. Find the limit:

a) $\lim_{x \rightarrow 1} \frac{\cos(\pi x) + 1}{(x-1)^2} =$

b) $\int_1^{\infty} \frac{\ln x}{x} dx =$

c) $\int_1^{\infty} \frac{dx}{x^5} =$

8. Find the Taylor expansion for $\int \frac{dx}{1+x^4}$ centered at $x = 0$. What is its radius of convergence?

9. Do the following series converge or diverge? Give your reasoning.

a) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

b) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

c) $\sum_{n=1}^{\infty} \frac{n}{n^3+n^2+1}$

10. Find the area enclosed by the curve given in polar coordinates by $r = 4 \sec \theta$ from $\theta = 0$ to $\theta = \pi/3$.

11. Here is the equation of an hyperbola:

$$2x^2 - 6y^2 + 10x - 12y = 92 .$$

Find the coordinates of its center and vertices, and the slopes of its asymptotes.

12. Solve the initial value problem:

$$y'' + 8y = e^{5x} , \quad y(0) = 4, y'(0) = 0 .$$