

Mathematics 1220-90 Summer, 2003, Final Examination

1. Find the integral : $\int \frac{\ln x}{x^2} dx$

2. Integrate $\int_1^4 (x^2 + 3x)\sqrt{x} dx$

3. Integrate : $\int \frac{u^2 + 1}{u^2(u - 1)} du$

4. Four years ago I invested \$10,000 in an account bearing continuously compounded interest. Today I have \$13,500. Assuming that the same interest rate continues into the future, when will my account have \$20,000?

5. Find the limit. Show your work.

a) $\lim_{x \rightarrow 4} \frac{\sin(\pi x)}{x^2 - 16} =$

b) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{2x^2} =$

c) $\lim_{x \rightarrow \infty} \frac{x^3}{e^x} =$

6. Do the integrals converge? If so, evaluate:

a) $\int_1^{\infty} \frac{dx}{1 + x^2}$

b) $\int_1^{\infty} \frac{dx}{1 + x}$

7. Find the sum of the series. If the series does not converge, just write “DIV”. Carefully note the limits of summation.

a) $\sum_{n=0}^{\infty} \frac{2^{n-1}}{3^n}$

b)
$$\sum_{n=3}^{\infty} \frac{1}{n(n-1)}$$

c)
$$\sum_{n=0}^{\infty} \frac{n}{3^{n-1}}$$

d)
$$\sum_{n=0}^{\infty} \frac{(12)^n}{n!}$$

8. Consider the hyperbola given by the equation $x^2 - 2y^2 - 2x + 12y = 138$.

a) What is its center? b) What is the distance between the its vertices?

9. Find the area of the region enclosed by the curve given in polar coordinates by $r = 2 \cos \theta$.

10. a) Find the general solution of the differential equation $y'' - 6y' + 5y = 0$.

b) Solve the initial value problem:

$$y'' - 6y' + 5y = 10 \quad , \quad y(0) = 0, y'(0) = 0 .$$