Calculus II
Final Exam, Spring 2003

1. Find the integrals:
a) $\int (e^{\sin x})^2 \cos x dx$

b) $\int x\sqrt{x-1}dx$

2. Integrate $\int \frac{t^2}{(t^2-1)(t-2)}dt$

3. Integrate $\int x \ln x dx$

4. A certain compound transforms from state A to state B at a (per minute) rate proportional to the concentration of B in the mixture:

$$\frac{dc_A}{dt} = -.02c_B \ ,$$

where c_A and c_B are the concentrations of A and B respectively (and, assuming no other material is present, $c_A + c_B = 1$). If at time $t = 0$ the mixture is 90% in state A how long will it take to be 10% A?

5. Find the limit. Show your work.
a) $\lim_{x\to 1} \frac{\ln x}{\sin(\pi x)} =$

b) $\lim_{x\to 0} \frac{xe^x}{e^{2x}-1} =$

c) $\lim_{x\to\infty} \frac{3x^6+7x^4}{2(x^3+1)^2} =$

6. Do the integrals converge? If so, evaluate:
a) $\int_0^\infty xe^{-x}dx$

b) $\int_2^\infty \frac{dx}{x(\ln x)^{25}}$

7. Do the series converge or diverge? Give a valid reason for your answer.
a) $\sum_{n=1}^{\infty} \frac{n^2+1}{(n+1)^3}$

b) $\sum_{n=1}^{\infty} \frac{\ln n}{2^n}$

c) $\sum_{n=1}^{\infty} \frac{(n!+1)^2}{((n+1)!)^2}$

8. Find the vertices of the conic given by the equation $4x^2 - y^2 + 8x - 4y + 12 = 0$.

9. Find the area of the region enclosed by the curve given in polar coordinates by $r = 2e^{\theta}$, $0 \le \theta \le 2\pi$ and the segment of the x axis between $x = 2$ and $x = 2e^{2\pi}$.

10. a) Find the general solution of the homogeneous differential equation $y'' - 3y' + 2y = 0$.

b) Find a particular solution of the homogeneous differential equation $y'' - 3y' + 2y = \sin x$.