

Mathematics 1220-90 Final Examination. December 10-11,2003

You must show your work. The answer alone is insufficient for any points.

1. Find the integrals:

a)
$$\int_2^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

b)
$$\int_0^2 \frac{x^2}{1+x^2} dx$$

2. Integrate
$$\int \frac{u+1}{u(u-1)} du$$

3. Integrate
$$\int x e^x dx$$

4. The population of Sourwater Canyon, New Mexico has been continuously decreasing at a steady rate for decades. Assuming continued decay at the same rate, if the population ten years ago was 8,000 and today it is 5,000, when will there be only 2 people left in Sourwater Canyon?

5. Find the limit. Show your work.

a)
$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-4} =$$

b)
$$\lim_{x \rightarrow 0} x \ln x =$$

c)
$$\lim_{x \rightarrow \infty} \frac{x^2}{(2x+1)^2} =$$

6. Find the integral

a)
$$\int_2^{\infty} \frac{dx}{x^{\frac{10}{9}}}$$

b)
$$\int_0^2 \frac{dx}{x^{\frac{9}{10}}}$$

7. The function $f(x)$ is defined for $-3 \leq x \leq 3$, and has the Maclaurin series at the origin:

$$f(x) = \sum_{n=0}^{\infty} \frac{(n+1)^2}{n!} x^n .$$

a) What is the radius of convergence of this series?

b) What is the Maclaurin series for $F(x) = \int_0^x f(t)dt$?

c) What is the Maclaurin series for $x^2 F(x)$?

8. Find the focus of the parabola given by the equation $x^2 - 8y + 2x + 17 = 0$.

9. Find the area of the region enclosed by the curve given in polar coordinates by $r = 2 \cos \theta \sqrt{\sin \theta}$, $0 \leq \theta \leq \pi/2$.

10. Find the general solution, for $x > 0$ of the differential equation

$$x \frac{dy}{dx} + \ln x = 0 .$$