Mathematics 1220 Calculus II, Examination 2, Feb 12, 14, 2004

Find all the integrals. Remember that definite integrals should have numerical answers. You MUST show your work.

1a.

$$\int (x+1)x^{12}dx = \int (x^{12}+x^{13})dx = \frac{x^{13}}{13}+\frac{x^{14}}{14}+C\ .$$

1b.

$$\int \ln(x^2)dx = \int 2\ln x dx\ .$$

Let $u = \ln x,\ dv = dx,\ du = dx/x,\ v = x$ and integrate by parts:

$$\int \ln(x^2)dx = 2(\int \ln x dx) = 2(x\ln x - \int dx) = 2(x\ln x - x + C)\ .$$

2.

$$\int \frac{dx}{x^2(x+2)}$$

Answer. We want to use partial fractions, so we begin with

$$\frac{1}{x^2(x+2)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+2} = \frac{Ax(x+2)+B(x+2)+Cx^2}{x^2(x+2)}\ .$$

Evaluate at the roots: for $x=0$, we get $1=2B$, so $B=1/2$; for $x=-2$, $1=4C$, so $C=1/4$. TO find A we have to equate the coefficients of $x^2:\ 0=A+C$, so $A=-1/4$. Thus, we get

$$\frac{1}{x^2(x+2)} = -\frac{1/4}{x}+\frac{1/2}{x^2}+\frac{1/4}{x+2}\ ,$$

so that

$$\int \frac{dx}{x^2(x+2)} = -\frac{1}{4}\ln x - \frac{1}{2x}+\frac{1}{4}\ln(x+2)+C = -\frac{1}{2x}+\frac{1}{4}\ln(\frac{x+2}{x})+C\ .$$

3.

$$\int_0^2 \frac{x}{1+x^4}dx$$

Answer. Recall that the integral of $(1+u^2)^{-1}du = \arctan u$. So, substitute $u=x^2,\ du = 2xdx$:

$$\int_0^2 \frac{x}{1+x^4}dx = \frac{1}{2}\int_0^4 \frac{du}{1+u^2} = \frac{1}{2}\arctan 4\ .$$

4. $$\int x(\sin x)dx$$

Answer. We want to integrate by parts, taking $u = x,\ dv = \sin x,\ du = dx,\ v = -\cos x$:

$$\int x(\sin x)dx = -x\cos x + \int \cos x dx = -x\cos x + \sin x + C\ .$$

5. $$\int_2^4 \frac{dx}{x(x-1)}$$

Answer. Use partial fractions to find

$$\frac{1}{x(x-1)} = \frac{1}{x-1} - \frac{1}{x}\ .$$

Then

$$\int_2^4 \frac{dx}{x(x-1)} = \int_2^4 \left(\frac{1}{x-1} - \frac{1}{x}\right)dx = \ln\left(\frac{x-1}{x}\right)\Big|_2^4$$

$$= \ln\frac{3}{4} - \ln\frac{1}{2} = \ln\frac{3}{2}\ .$$