Mathematics 1220 Calculus II, Examination 2, Sep 25,27, 2003

Find all the integrals. Remember that definite integrals should have numerical answers. You MUST show your work.

1a. $$\int \frac{dx}{(1+x)\sqrt{x}}$$

Solution. Make the substitution $u = x^{1/2},\ du = (1/2)x^{-1/2}dx$:

$$\int \frac{dx}{(1+x)\sqrt{x}} = 2\int \frac{du}{1+u^2} = 2\arctan u + C = 2\arctan(\sqrt{x}) + C\ .$$

1b. $$\int \frac{2+x}{1+x}dx$$

Solution. Rewrite $2+x = 1+x+1$ so that $(2+x)/(1+x) = 1+1/(1+x)$. Then

$$\int \frac{2+x}{1+x}dx = \int (1+\frac{1}{1+x})dx = x + \ln(1+x) + C\ .$$

2. $$\int e^x x dx$$

Solution. Integrate by parts: $u = x,\ du = dx,\ v = e^x,\ dv = e^x dx$. Then

$$\int e^x x dx = xe^x - \int e^x dx = xe^x - e^x + C\ .$$

3. $$\int_1^2 \frac{x^2-4x+1}{x(x-4)^2}dx$$

Solution. We look for the partial fractions representation:

$$\frac{x^2-4x+1}{x(x-4)^2} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{(x-4)^2}\ .$$

Combining the right hand side over a common denominator, we can equate the numerators:

$$x^2 - 4x + 1 = A(x-4)^2 + Bx(x-4) + Cx\ .$$

Set $x = 0$ to get $1 = A(-4)^2$, or $A = 1/16$. Now, set $x = 4$ to get $16 - 16 + 1 = 4C$, or $C = 1/4$. To find the value of B we compare the coefficients of x^2 on both sides. On the left we have 1, and on the right $A + B$. This gives $1 = A + B = 1/16 + B$, so $B = 15/16$. Thus

$$\frac{x^2 - 4x + 1}{x(x-4)^2} = \frac{1}{16}\frac{1}{x} + \frac{15}{16}\frac{1}{x-4} + \frac{1}{4}\frac{1}{(x-4)^2} \ ,$$

$$\int_1^2 \frac{x^2 - 4x + 1}{x(x-4)^2} dx = \Big[\frac{1}{16}\ln|x| + \frac{15}{16}\ln|x-4| - \frac{1}{x-4}\Big]_0^2$$

$$= \frac{1}{16}(\ln 2 - \ln 1) + \frac{15}{16}(\ln 2 - \ln 3) - (\frac{1}{2} - \frac{1}{3}) = \ln 2 - \frac{15}{16}\ln 3 - \frac{1}{6} \ .$$

4a. **Solution** $$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C = -\arccos x + C \ .$$

4b. $$\int \frac{xdx}{1+4x^2} =$$

Solution. Let $u = 1 + 4x^2,\ du = 8xdx$. Then

$$\int \frac{xdx}{1+4x^2} = \frac{1}{8}\int \frac{du}{u} = \frac{1}{8}\ln u + C = \frac{1}{8}\ln(1+4x^2) + C \ .$$

5. $$\int \ln x dx$$

Solution. Integrate by parts. Let $u = \ln x,\ du = dx/x,\ dv = dx,\ v = x$:

$$\int \ln x dx = x\ln x - \int x\frac{dx}{x} = x\ln x - x + C \ .$$